Land-Price Dynamics and Macroeconomic Fluctuations with CES Household Preferences

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摘　要

This paper studies land-price dynamics and macroeconomic fluctuations by introducing a general household's preference into Liu et al. (2013) that allows for the intertemporal and the intratemporal elasticities of substitution (ES) to deviate from unity. When the intertemporal ES is smaller than unity and also smaller than the intratemporal ES, in response to a positive housing demand shock, we find that consumption increases and comoves with land prices and business investment consistent with the estimated evidence from a BVAR model. Moreover, the shares of investment, labor hours, and output explained by the housing demand shock are different from those in Liu et al. (2013). We estimate alternative models to fit the time series data and also estimate the values of the two ESs within the structural model. We find the result in favor of the model with the household's preference featuring a complementary relationship across periods but a substitutable relationship within a period.

Keywords: land prices, housing demand shocks, CES preferences, collateral constraints

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1. Introduction

The financial crisis of 2007-2008 propelled the U.S. and world economies into the most severe global recession since the Great Depression. Triggered by the sudden and severe slump of the U.S. housing market, the financial crisis shrank asset values tied to U.S. real estate. This came with a sharp decline in housing and land prices and a harsh collapse in business investment, leaving a stark decrease in employment, output and consumption. The crisis has sparked substantial interest in what drives housing prices and how they affect macroeconomies.[1] In a recent paper, Liu et al. (2013) introduced land as a collateral asset in firms' credit constraints, as in Kiyotaki and Moore (1997), and found that a positive shock to housing demand can generate a mechanism that amplifies and propagates the shock through the strong joint dynamics between land prices and business investment. In particular, their estimation indicated that the housing demand shock alone accounts for about 90% of the fluctuations in land prices, along with large fluctuations in investment, output, and labor hours. The empirical macroeconomic literature has suggested that housing demand shocks are the primary driving force for the fluctuation of the house price (Davis and Heathcote, 2007). Liu et al. (2013)'s paper is valuable in identifying the interactions between the housing market and the macroeconomy via the collateral and the land reallocation channels that improve policy making. However, their theoretical impulse responses have a comovement puzzle: their consumption decreases in response to a positive housing demand shock, which is inconsistent with the evidence estimated from a Bayesian vector autoregression (BVAR) model. Their estimates indicate that consumption increases and comoves with land prices, investment, and hours following a positive shock to the land-price series (cf. Liu et al. 2013, Figure 2).

The comovement puzzle arises from the household's utility in Liu et al. (2013), which is log-separable in consumption and durable land services. As a result, the intertemporal *elasticity of substitution* (hereafter, ES) of consumption bundles over time and the intratemporal ES between consumption and land services in a period of time both are unity. As Hall (1988) pointed out, intertemporal substitution in consumption is a central element of most modern macroeconomic models. The quantitative importance of the effects of changes in

[1] See Iacoviello and Neri (2010), Mian et al. (2013), Adelino et al. (2015), and Berger et al. (2018), among others.

various policies depends on the magnitude of the intertemporal ES. Moreover, as Ogaki and Reinhart (1998) put it, intratemporal substitution is vital when there are consumer durables. The intratemporal substitution between nondurables and durables is important not only for understanding the effects of the housing demand shock on business cycles but also for other issues like asset pricing and asset risk premia.

In this paper, we introduce a general household's utility into Liu et al. (2013) and investigate, in response to a positive housing demand shock, whether the comovement puzzle can be resolved and if the striking results of Liu et al. (2013) are modified. Specifically, the household's utility for consumption and land services is a constant-elasticity-of-substitution (CES) function, as in Barsky et al. (2003, 2007), Monacelli (2009), and Chen and Liao (2014). The utility has two parameters, representing the intertemporal ES and the intratemporal ES, which reduces to Liu et al. (2013)'s utility if both ESs go to unity. We establish that the utility can resolve the comovement puzzle if the intertemporal ES is less than unity and smaller than the intratemporal ES. The reason goes as follows: following a positive housing demand shock and thus, a land price increase, when the intertemporal ES is less than unity, agents are less willing to substitute away from current toward future consumption. If the intertemporal ES is smaller than the intratemporal ES, the land price increase causes land services to be substituted away toward more current consumption. As a result, current consumption increases and comoves with land prices and business investment, so the comovement puzzle is resolved.

An intertemporal ES less than unity and smaller than the intratemporal ES is consistent with the existing estimates in the literature. Ogaki and Reinhart (1998) and Piazzesi et al. (2007) estimated that the intertemporal ES is statistically and significantly less than unity and the intratemporal ES is statistically and significantly larger than unity, while Yogo (2006), and more recently Li et al. (2016), found that the intertemporal ES is statistically and significantly smaller than unity and less than the intratemporal ES. Flavin and Nakagawa (2008) estimated the intertemporal ES to be about a half, while Bajari et al. (2013) estimated the intratemporal ES to be statistically and significantly larger than unity. These estimates indicate that the intertemporal ES is smaller than unity, and the intratemporal ES is greater than the intertemporal ES and may be larger than unity.

To estimate the model, we fit the log-linearized equilibrium system to the same data as in Liu et al. (2013). We use the Bayesian method to estimate structural parameters and shock parameters in the model. The estimated parameters of our model suggest that the estimation of Liu et al. (2013) is in general robust. However, because of the joint effects of intertemporal and intratemporal ESs deviating from unity, the impulse responses and the amplification effects on key macroeconomic variables are different. Our study focuses on two baseline cases for the intertemporal ES. One of the baselines sets the intertemporal ES at unity, as in Liu et al. (2013), and the other sets the intertemporal ES at less than unity consistent with the existing estimates. Under each of the two baselines, we study impulse responses under three values of the intratemporal ES. One value is unity, as in Liu et al. (2013), one value is larger than the baseline intertemporal ES at unity, and one value is smaller than the other baseline intertemporal ES less than unity. The first two values of the intratemporal ES lie within the range of the existing estimates, and the last value is used as a control group. Finally, other then these parameter values set from the outside, we also estimate the values of the two ESs within the structural model and let the data and the model decide what the estimates should be.

The impulse responses indicate that, as in Liu et al. (2013), a positive housing demand shock leads to persistent increases not only in land prices but also in business investment, since increases in land prices activate the financial multiplier that interacts land prices with business investment in propagating the shock, and the housing demand shock alone accounts for the lion’s share of the fluctuations in the land price.

There are three different results. First, consumption increases and comoves with land prices and business investment if the intertemporal ES is less than unity and smaller than the intratemporal ES. The reason is that, with the intertemporal ES being smaller than unity, the intertemporal consumption smoothing effect is large, and, with the intertemporal ES being less than the intratemporal ES, the intertemporal consumption smoothing effect dominates the intratemporal consumption smoothing effect. As a result, a positive housing demand shock raises land prices by more. A higher land price causes land services to be substituted away toward current consumption, and thus consumption increases and comoves with land prices and investment.

Next, in comparing the relative importance of the housing demand shock and other structural shocks in driving the impulse responses, when intertemporal and intratemporal ESs are different from unity, variance decompositions indicate that the fractions of the fluctuation in macroeconomic variables accounted for by the housing demand shock are different from those in Liu et al. (2013).

Finally, we estimate alternative models to fit the data as in Liu et al. (2013) and estimate the values of the two ESs within the structural model. We find the results in support of the model with a household's preference featuring a complementary relationship for consumption bundles across periods and a substitutable relationship for consumption and land services within a period.

Our finding that consumption increases and comoves with land prices is in line with the result uncovered by Campbell and Cocco (2007), and more recently, by Kaplan et al (2020). These papers used micro data and studied how house price fluctuations affected households' consumption. They found that an increase in house prices increased consumer spending.

Our paper is related to Gong et al. (2017), which extended the household utility in Liu et al. (2013) to a non-separable utility between consumption and labor, and a log utility of land services. Gong et al. (2017) focused on comparing the two cases of the intertemporal ES that are equal to or less than unity. There are two main differences. First, the model is different. As in Liu et al. (2013), our utility has an infinite Frisch elasticity of labor supply (hereafter, ELS), but Gong et al. (2017) has a finite Frisch ELS. Our household utility is non-separable in consumption and land services with a general intratemporal ES, but consumption is separable from land services in Gong et al. (2017). In particular, the values of the two ESs in our model are estimated within the structural model, but the value of the intertemporal ES in Gong et al. (2017) is set from the outside. Next, the impulse responses are different. In response to a positive housing demand shock, our consumption increases and comoves with land prices and investment, but in Gong et al. (2017), consumption decreases and does not comove with land prices. Moreover, in the case when the intertemporal ES is less than unity, the fractions of the fluctuations in land prices and key variables accounted for by the housing demand shock are larger in our model but smaller in Gong et al. (2017) than those in Liu et al. (2013), wherein the intertemporal ES is unity. Latter result emerges, since the finite Frisch ELS in Gong et al. (2017) by itself leads to lower labor supply, investment and collateral capital, which lowers borrowing. Thus, the degree to which the entrepreneur's credit constraint is relaxed is lower and a smaller dynamic financial multiplier emerges.

Our paper is related to papers by Barsky et al. (2003, 2007), Monacelli (2009), and Chen and Liao (2014). Like our paper, these existing papers studied consumer nondurables and durables (or land services). The difference is that we analyze the impulse responses of the housing demand shock, while these existing papers study the impulse responses of the monetary policy shock.

Finally, our paper follows Liu et al. (2013) and sets up two types of agents, a patient household and an impatient entrepreneur that uses land as a collateral for loans. There is a strand of recent dynamic stochastic general equilibrium literature with two types of agents where one of the types uses land as collateral for loans (Iacoviello, 2005; Iacoviello and Neri, 2010; Justiniano et al., 2015; Favilukis et al., 2017). Our model is different from the literature, as we have no impatient households and it is firms that are credit constrained. Our paper is also broadly related to the papers that studied the amplification effect through the borrowing constraint. On this, the seminal work is Kiyotaki and Moore (1997). See also papers by Kocherlakota (2000), Cordoba and Ripoll (2004), and Cao and Nie (2017), which analyzed the quantitative significance of the amplification through the borrowing constraint.

The organization is as follows. Section 2 sets up the model, while Section 3 is the estimation strategy. Section 4 studies the impulse responses of positive housing demand shocks and the role of different intertemporal and intratemporal elasticities of substitutions. Section 5 analyzes relative importance of different structural shocks in terms of variance decompositions. Finally, Section 6 is the concluding remarks.

2. The model

Our model is otherwise identical to that of Liu et al. (2013) except for a general household's preference. The economy has a representative household and a representative entrepreneur. The household consumes goods and land services (housing) and supplies labor, while the entrepreneur consumes consumption goods only. The entrepreneur produces final goods using labor, capital, and land, and needs external financing for investment. Due to imperfect contract enforcement, the borrowing capacity is constrained by the value of collateral assets, consisting of land and capital. As in Liu et al. (2013), we assume that the household is more patient than the entrepreneur, so that the entrepreneur's collateral constraint is binding in and near the steady-state equilibrium. The supply of land is fixed.

2.1 The representative entrepreneur

As in Liu et al. (2013), the entrepreneur's utility is given by

$$E\ \sum_{t=0}^{\infty}\beta^{t}\{\log\ (C_{e,t}-\gamma_{e}C_{e,t-1})\}, \tag{1a}$$

where $C_{e,t}$ is entrepreneur's consumption, γ_e is the degree of entrepreneur's habit persistence, and $\beta\in(0, 1)$ is the discount factor.

The entrepreneur produces final goods with the production technology given by

$$Y_t = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{e,t}^{1-\alpha}, \tag{1b}$$

where Y_t denotes output, $L_{e,t-1}$ is land, K_{t-1} is capital, and $N_{e,t}$ is labor input. Parameters $\alpha \in (0, 1)$ and $\phi \in (0, 1)$ measure the output elasticities of these production factors.

As in Liu et al. (2013), the total factor productivity Z_t consists of a permanent component Z_t^p and a transitory component v_t such that $Z_t = Z_t^p v_{z,t}$. The permanent component Z_t^p follows $Z_t^p = Z_{t-1}^p\lambda_{zt}$, where the growth rate λ_{zt} and the transitory component $v_{z,t}$ follow the stochastic process given, respectively, by

$$ln\lambda_{z,t} = (1-\rho_z)ln\bar{\lambda}_z + \rho_z ln\lambda_{z,t-1} + \sigma_z\varepsilon_{z,t}, \tag{2a}$$

$$lnv_{z,t} = \rho_{vz}lnv_{z,t-1} + \sigma_{vz}\varepsilon_{vz,t}. \tag{2b}$$

In (2a) and (2b), $\bar{\lambda}_z$ is the steady-state value of λ_{zt}. Parameters ρ_z and $\rho_{vz} \in (-1, 1)$ are the degree of persistence, σ_z and σ_{vz} are the standard deviations, and the innovations $\varepsilon_{z,t}$ and $\varepsilon_{vz,t}$ are independent and identically distributed (*i.i.d.*) standard normal processes.

The entrepreneur faces the flow of funds constraint given by

$$C_{e,t} + q_{l,t}(L_{e,t} - L_{e,t-1}) - \frac{B_t}{R_t} = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{e,t}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{e,t} - B_{t-1}, \tag{3}$$

where I_t is investment, $q_{l,t}$ is the land price, B_{t-1} is the amount of matured debts, R_t is the gross real interest rate, w_t is the real wage rate, and B_t /R_t is the value of new debts.

As in Greenwood et al. (1997), there is the investment-specific technology change Q_t. Following Liu et al. (2013), we assume that $Q_t = Q_t^p v_{q,t}$, in which the permanent component Q_t^p follows $Q_t^p = Q_{t-1}^p \lambda_{qt}$, where the growth rate λ_{qt} and the transitory component $v_{q,t}$ follow the stochastic process given, respectively, by

$$ln\lambda_{q,t} = (1-\rho_q)ln\bar{\lambda}_q + \rho_q ln\lambda_{q,t-1} + \sigma_q \varepsilon_{q,t}, \quad (4a)$$

$$lnv_{q,t} = \rho_{vq} lnv_{q,t-1} + \sigma_{vq}\varepsilon_{vq,t}, \quad (4b)$$

where $\bar{\lambda}_q$ is the steady-state value of λ_{qt}, and the parameters ρ_q and $\rho_{vq} \in (-1, 1)$ are the degree of persistence, while σ_q and σ_{vq} are the standard deviations, and the innovations $\varepsilon_{q,t}$ and $\varepsilon_{vq,t}$ are *i.i.d.* standard normal processes.

The entrepreneur is endowed with K_{t-1} units of capital and $L_{e,t-1}$ units of land initially. Capital is accumulated from investment that follows the law of motion given by

$$K_t = (1-\delta)K_{t-1} + [1 - \frac{\Omega}{2}(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I)^2]I_t, \quad (5)$$

where δ is the depreciation rate, $\bar{\lambda}_I$ is the steady-state growth rate of investment, and $\Omega>0$ is the adjustment cost parameter.

The loan market is imperfect, and collateral is required in order to take out loans. The entrepreneur faces the following credit constraint

$$B_t \leq \varsigma_t E_t[q_{l,t+1}L_{e,t} + q_{k,t+1}K_t], \quad (6)$$

where $q_{k,t+1}$ is the shadow price of capital in unit of final goods, and ς_t is interpreted as a collateral shock.

Following Kiyotaki and Moore (1997), we interpret this type of credit constraint as reflecting the problem of costly contract enforcement. The credit constraint implies that, if the entrepreneur fails to repay the debt in the next period, the creditor can seize the collateral assets, which is the value of land and accumulated capital in the next period. As it is costly to liquidate the seized land and capital stock, the creditor can recover up to a fraction ς_t of the total value of collateral assets.

Following Liu et al. (2013), ς_t follows the stochastic process given by

$$ln\varsigma_t = (1-\rho_\varsigma)ln\bar{\varsigma} + \rho_\varsigma ln\varsigma_{t-1} + \sigma_\varsigma \varepsilon_{\varsigma,t}, \quad (7)$$

where $\bar{\varsigma}$ is the steady-state value of ς_t, and $\rho\varsigma \in (-1, 1)$ is the persistent parameter, while σ_ς is the standard deviation, and the innovations $\varepsilon_{\varsigma,t}$ is an *i.i.d.* standard normal process.

2.2 The representative household

Different from Liu et al. (2013), the household's discounted utility function is given by

$$E\sum_{t=0}^{\infty}\beta^t A_t[U(C_{h,t}, L_{h,t}) - \psi_t N_{h,t}], \quad (8)$$

where $U(C_{h,t}, L_{h,t}) = \frac{1}{1-1/\eta}\left(u(C_{h,t}, L_{h,t})\right)^{\frac{1-1/\eta}{1-1/\zeta}}$

and $u(C_{h,t}, L_{h,t}) = (1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{1-1/\zeta} + \chi\left(L_{h,t}^{\varphi_t}\right)^{1-1/\zeta}.$

The household derives utility from consumption $C_{h,t}$, (durable) land services $L_{h,t}$, and labor hours $N_{h,t}$, wherein consumption and land services in period t are aggregated to the consumption bundle $u(C_{h,t}, L_{h,t})$ in a CES function, with χ denoting the relative weight between consumption and land services.[2] The household also obtains a negative utility from supplying labor hours, which gives an infinite Frisch ELS, as in Liu et al. (2016). The parameter γ_h is the degree of household's habit persistence.

Following Liu et al. (2016), consumption is scaled by the growth factor $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha}]^{\frac{1}{1-(1-\phi)\alpha}}$ in order to be consistent with the balanced growth, where Z_t is the total factor productivity and Q_t is the investment-specific technology. In (8), A_t is the household's patience factor and evolves as $A_t = A_{t-1}(1+\lambda_{a,t})$, where $\lambda_{a,t}$ represents a shock to the household's patience factor. Moreover, φ_t is a shock to the household's demand for land services, which is also labeled as the housing demand shock, and ψ_t is a shock to the labor supply. The

2 As in Liu et al. (2013), we will interchange the term housing services with land services.

stochastic processes of those shocks are the same as those in Liu et al. (2013), given as follows.

$$ln\lambda_{a,t} = (1-\rho_a)ln\bar{\lambda}_a + \rho_a ln\lambda_{a,t-1} + \sigma_a \varepsilon_{a,t}, \tag{9a}$$

$$ln\varphi_t = (1-\rho_\varphi)ln\bar{\varphi} + \rho_\varphi ln\varphi_{t-1} + \sigma_\varphi \varepsilon_{\varphi,t}, \tag{9b}$$

$$ln\psi_t = (1-\rho_\psi)ln\bar{\psi} + \rho_\psi ln\psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t}, \tag{9c}$$

where $\bar{\lambda}_a,\ \bar{\varphi},\ \bar{\psi} > 0$ are steady-state values of $\lambda_{a,t}, \varphi_t, \psi_t$, while ρ_a, ρ_φ, $\rho_\psi \in (-1, 1)$ are persistence parameters, and σ_a, σ_φ, $\sigma_\psi > 0$ are the standard deviations of the innovation. The innovations $\varepsilon_{a,t}$, $\varepsilon_{\varphi,t}$, $\varepsilon_{\psi,t}$ are *i.i.d.* standard normal processes.

The utility function U_t has two parameters η and ζ. The parameter η is the intertemporal ES between consumption bundles across periods. For a higher value of η, the households are more willing to substitute consumption bundles over time. The bundles are perfect substitutes over time as $\eta \to \infty$, are perfect complements as $\eta \to 0$, and are the Cobb–Douglas function as $\eta \to 1$.[3] The parameter ζ represents the intratemporal ES between consumption and land services within a period. For a higher value of ζ, the households are more willing to substitute one for the other. The two goods become perfect substitutes within a period as $\zeta \to \infty$ and perfect complements as $\zeta \to 0$. Taking the limit as $\zeta \to 1$ yields the Cobb–Douglas function. In the case if $\zeta = \eta$, the utility is separable in consumption and land services.

These two parameters η and ζ affect the cross partial derivative of the utility function U_t with respect to the two goods, which is[4]

$$U_{C_h L_h} \equiv \frac{\partial}{\partial L_{h,t}}\left(\frac{\partial U_t}{\partial C_{h,t}}\right) = (\frac{1}{\zeta} - \frac{1}{\eta})(1-\chi)\chi\varphi_t(u_t)^{\frac{1-1/\eta}{1-1/\zeta}-2}(L_{h,t}^{\varphi_t(1-\frac{1}{\zeta})-1})(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t})^{-\frac{1}{\zeta}}, \tag{10}$$

which informs about the relative strength of the intratemporal and intertemporal tradeoffs.

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[3] We use standard Hicksian language here. Two goods are substitutes if and only if $\eta > 1$ and complements if and only if $\eta < 1$. The property can be inferred from data on quantities and relative prices, and has nothing to do with agent's intertemporal concern for smoothing consumption.

[4] Some papers refer to $U_{CL} < 0$ as the case in which C and L are substitutes, while $U_{CL} > 0$ is the case in which these two goods are complements. We refrain from such a language here, since the cross partial derivative of the felicity function captures both intertemporal and intratemporal tradeoffs.

The sign of the cross partial derivative is determined by $(\eta - \zeta)$, which is positive if $(\eta - \zeta) > 0$, and negative if $(\eta - \zeta) < 0$. The special case $\eta = \zeta = 1$ and yields the household's utility in Liu et al. (2013) and $(\eta - \zeta) = 0$. In particular, when $(\eta - \zeta) < 0$, the intratemporal tradeoff is larger than the intertemporal tradeoff and then, a higher housing price tends to substitute away from land services toward consumption.

The existing literature has estimated the intertemporal ES η to be smaller than unity and the intratemporal ES ζ to be on average larger than unity. Using a homothetic preference to estimate the intertemporal ES and the intratemporal ES, Ogaki and Reinhart (1998) found $\eta \in [0.32, 0.45]$ and $\zeta = 1.17$ and Yogo (2006) obtained $\eta = 0.02$ and $\zeta \in [0.52, 0.87]$, while Piazzesi et al. (2007) attained $\eta \in [0.06, 0.20]$ and $\zeta \in [1.05, 1.25]$ and Flavin and Nakagawa (2008) discovered $\eta \in [0.54, 0.55]$. Recently, Bajari et al. (2013) estimated $\zeta = 4.55$, and Li et al. (2016) attained $(\eta, \zeta) = (0.14, 0.49), (0.16, 0.81), (0.83, 1.69)$. Thus, the estimated values $\eta \in [0.02, 0.83]$ are less than unity and also are smaller than the estimated value of ζ, which is on average larger than unity.

The household faces the flow budget constraint given by

$$C_{h,t} + q_{l,t}(L_{h,t} - L_{h,t-1}) + \frac{S_t}{R_t} \leq w_t N_{h,t} + S_{t-1}, \tag{11}$$

where S_t is the risk-free bond. In the initial period, the household is endowed with $L_{h,t-1}>0$ units of land and $S_{-1}>0$ units of the risk-free bond.

The household chooses $C_{h,t}$, $L_{h,t}$, $N_{h,t}$, and S_t to maximize the expected lifetime utility in (8) subject to (11) and the borrowing constraint $S_t \geq -\bar{S}$ for some large number $\bar{S}$.

2.3 Equilibrium

There are four markets, including the markets for final goods, land, labor, and loans. All markets clear in the competitive equilibrium. First, the final goods market clearing condition is

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{12a}$$

where $C_t = C_{h,t} + C_{e,t}$ is aggregate consumption. Next, the land market clearing condition is

$$L_{h,t} + L_{e,t} = \bar{L}. \tag{12b}$$

Moreover, the clearing condition for the labor market is

$$N_{e,t} = N_{h,t} \equiv N_t. \tag{12c}$$

Finally, the market clearing condition for loans is

$$S_t = B_t. \tag{12d}$$

A competitive equilibrium is sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{h,t}, C_{e,t}, I_t, N_{h,t}, N_{e,t}, L_{h,t}, L_{e,t}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that given the sequence of prices, (i) the allocations maximize the household's problem, and (ii) the allocations solve the entrepreneur's problem, and (iii) all the markets clear.

3. Estimation

We take a log-linearization of the equilibrium system around the steady state. Following Liu et al. (2013), we use the Bayesian approach to fit the log-linearized equilibrium system to the same six quarterly U.S. time series as used by Liu et al. (2013): the relative price of land ($q_{l,t}$), the inverse of the quality-adjusted relative price of investment (Q_t), real consumption per capita (C_t), real investment per capita in consumption units (I_t), real nonfinancial business debts per capita (B_t), and per capita hours worked (N_t). All these series are constructed in line with the corresponding series in Greenwood, Hercowitz, and Krusell (1997), Cummins and Violante (2002), and Davis and Heathcote (2007). The sample covers the period from 1975: Q1 to 2010: Q4.

In the Bayesian estimation, a system of measurement and transition equations links observable variables to state variables. By setting prior distributions and updating the joint distribution through the information contained in the observed data, the posterior distribution of the parameter set θ can be well approximated by some Markov chain Monte Carlo (MCMC) algorithm, and eventually the value of the parameter set is obtained by maximizing the likelihood function. Yet, with binding credit constraints, the posterior kernel is filled with narrow but twisty ridges and local peaks. Thus, it is difficult not only to find the mode of the posterior distribution but also to uncover the posterior mode of the built-in optimizing methods in the popular Dynare software.

Our optimization routine in estimating structural parameters and shock parameters is the same as the one used in Liu et al. (2013), which is based on Sims et al. (2008).[5] With an initial guess of the values of structural parameters and shock parameters, we use a combination of a constrained optimization algorithm and an unconstrained Broyden–Fletcher–Goldfarb–Shanno optimization algorithm to find a local peak. Then, the local peak is used to simulate a long sequence of MCMC posterior draws. These draws are then treated as different starting points in order for the optimization routine to find a potentially higher peak. We iterate this process, until the highest peak is found.

Our parameters are partitioned into three subsets, which are the structural parameters on which we have agnostic priors, the structural parameters for which we have the steady-state relations to construct informative priors, and the parameters which describe the shock processes. The prior distributions of these parameters are the same as those in Liu et al. (2013). First, we employ the steady-state values to calibrate the values of $\{\alpha, \bar{\theta}, \bar{\psi}\}$.[6] Next, we apply the Bayesian method to estimate the structural parameters on which we have agnostic priors $\{\gamma_h, \gamma_e, \Omega, g_\gamma, \bar{\lambda}_q\}$. Then, we identify the structural parameters for which we have the steady-state relations to construct informative priors $\{\beta, \bar{\lambda}_a, \bar{\varphi}, \phi, \delta\}$. Finally, we adopt agnostic priors for the persistence and standard deviations of the shock processes $\{\rho_i, \sigma_i\}$ for the eight shock parameters $i \in \{a, z, v_z, q, v_q, \varphi, \psi, \theta\}$.

The intertemporal ES is set at $\eta = 1$ in Liu et al. (2013). In addition to the baseline case $\eta = 1$, since the existing estimated values $\eta \in [0.02, 0.83]$ are smaller than unity, we will also explore the baseline case $\eta < 1$. The value $\eta = 0.5$ has been used frequently in the literature. Moreover, Liu et al. (2013) set the intratemporal ES to be $\zeta = 1$. In addition, given that the estimated ζ value is larger than η and also is on average greater than unity, we also allow for

5 The optimization routine in Sims et al. (2008) is coded in C/C++, which is downloadable at http://www.tzha.net/code. Compared with other optimization routines in Dynare 4.2, the optimization routine used in Liu et al. (2013) is efficient and can find the posterior mode. See the Appendix in Liu et al. (2013) for description of the data and the prior distributions.

6 As in Liu et al. (2013), the values of α and $\bar{\theta}$ are fixed at 0.3 and 0.75 in accord with the data, respectively, and the values of $\bar{\psi}$ is adjusted so that the steady-state market hours are about 25% of time endowment.

the case of the ζ value to be larger than 1, and set the value $\zeta = 1.2$. Moreover, as the sign of $(\zeta - \eta)$ matters for the cross partial derivative of the utility function with respect to consumption and land services, we also consider $\zeta = 0.4$ as a control group. Then, both cases $(\zeta - \eta) > 0$ and $(\zeta - \eta) < 0$ emerge, when the baseline is $\eta = 1$ and when the baseline is $\eta = 0.5$. Tables 1 and 2 are the estimated parameter values.

Case 1. $\eta = 1$

In this baseline case $\eta = 1$, we use the Bayesian approach to estimate structural parameters and shock parameters for these three sets of the value: $\zeta = 1.2$, 1 and 0.4. Tables 1a–1b report the prior distributions and the estimated posterior modes for structural parameters and shock parameters, respectively.

Case 2. $\eta = 0.5$

In this baseline case $\eta = 0.5$, we also use the Bayesian approach to estimate structural parameters and shock parameters for these three sets of the value: $\zeta = 1.2$, 1 and 0.4. Tables 2a–2b report the prior distributions and the estimated posterior modes for structural parameters and shock parameters, respectively.

4. Impulse responses of a positive housing demand shock

With the household's utility function different from that in Liu et al. (2013), the impulse responses of the housing demand shock are theoretically different. This section analyzes the impulse responses on the business cycles of land prices and key macroeconomic variables starting with the case $\eta = 1$, followed by the case $\eta = 0.5$. We investigate how different values of the intertemporal ES and the intratemporal ES affect the fluctuations in land prices and other variables in response to a positive housing demand shock.

4.1 Impulse responses of a positive housing demand shock when $\eta = 1$

Under the baseline $\eta = 1$, when there is a change in the interest rate, the intertemporal substitution effect is equal to the income effect. Now, we perform the impulse responses of an increase in the housing demand shock by one standard deviation under $\eta = 1$.

The impulse responses under ζ = 1 in In Table 1 replicates exactly those in Liu et al. (2013, Figure 4) when the household's utility is logarithmic with both elasticities equal to 1. In response to a positive housing demand shock, the household increases the land service and decreases consumption on impact. The increase in the land price propagates the shock and, by way of the expansions of net worth and the entrepreneur's borrowing constraint, triggers the dynamic financial multiplier through interactions between the land price and investment. With a logarithmic utility, the entrepreneur would smooth consumption over time by investing part of the loans, and this intertemporal smoothing incentive is reinforced by habit persistence. The entrepreneur's habit persistence dampens consumption and increases investment in responses to a shock that raises the land price. As a result, investment, labor hours and output increase, but aggregate consumption decreases on impact and thus, does not co-move with other aggregate variables.

The Liu et al. (2013) model corresponds to the case when the household's willingness to substitute consumption for land services in a period ζ =1 is equal to the willingness to substitute consumption bundles over time η = 1. Under given η = 1, Figure 1 also reports the impulse responses of two other cases of intratemporal ESs at ζ = 1.2 and 0.4. Now, the household's willingness to substitute consumption for land services in a period for the case ζ =1.2 and 0.4 is, respectively, larger and smaller than the willingness to substitute consumption bundles over time, which is η = 1.

For other two cases ζ = 1.2 and 0.4, Figure 1 indicates that increases in investment, labor hours and output are about the same as those in Liu et al. (2013)'s case (η, ζ) = (1, 1). The difference is, compared with the case (η, ζ) = (1, 1), land prices increase by more for the case (η, ζ) = (1, 0.4) and less for the case (η, ζ) = (1, 1.2), and aggregate consumption decreases by more for the case (η, ζ) = (1, 0.4).

The main problem with this baseline η = 1 is that, no matter whether the intratemporal ES is ζ = 1.2, 1, or 0.4, aggregate consumption decreases on impact. Such results are inconsistent with the evidence estimated from the BVAR model in Liu et al. (2013), which displays the comovement of consumption, land prices, business investment, and labor hours following a positive shock to the land-price series.

4.2 Impulse responses of a positive housing demand shock when $\eta = 0.5$

We turn to the other baseline case $\eta = 0.5$. In this baseline, the intertemporal substitution effect is smaller than the income effect. This implies that the household is less willing to smooth consumption bundles over time than the model with the baseline $\eta = 1$. Figure 2 reports the impulse responses of a one-standard deviation increase in the housing demand shock under the baseline $\eta = 0.5$ for the intratemporal ES at $\zeta = 1$ and two other intratemporal ESs at $\zeta = 1.2$ and 0.4. Now, the household's willingness to substitute consumption for land services in a period in the case $\zeta = 1$ and 1.2 is larger, but the willingness to substitute consumption in a period in the case $\zeta = 0.4$ is smaller, than the willingness to substitute consumption over time, which is $\eta = 0.5$.

Figure 2 indicates that, in response to a positive housing demand shock, aggregate consumption increases on impact for cases $\zeta = 1$ and 1.2. While aggregate consumption decreases on impact for the case $(\eta, \zeta) = (0.5, 0.4)$, the decrease is smaller than the corresponding case $(\eta, \zeta) = (1, 0.4)$ in Figure 1. The land price increases for all three cases $\zeta = 1.2$, 1, and 0.4, but, different from the case in Figure 1, the land price increases the highest in the case $\zeta = 1.2$, followed by $\zeta = 1$, and then $\zeta = 0.4$. As the increase in land price affects the entrepreneur's credit constraints and propagates the housing demand shock, the entrepreneur's investment increases to the largest in the case $\zeta = 1.2$, followed by $\zeta = 1$, and then $\zeta = 0.4$. As investment increases, labor hours also increase, as these inputs are complements in production. As a result, output increases the largest in the case $\zeta = 1.2$, followed by $\zeta = 1$, and then $\zeta = 0.4$. Hence, for cases $\zeta = 1$ and 1.2, aggregate consumption, land prices, business investment, and labor hours comove in response to a positive housing demand shock, which is consistent with the evidence estimated from the BVAR model in Liu et al. (2013).

4.3 Comparisons of the impulse responses between the baselines $\eta = 1$ and 0.5

Under each of the two intertemporal smoothing baselines $\eta = 1$ and $\eta = 0.5$, the former two subsections have compared the impulse responses for different intratemporal smoothing values, $\zeta = 1.2$, 1, and 0.4. This subsection takes a different perspective. Under each of the three intratemporal smoothing values, $\zeta = 1.2$, 1, and 0.4, we compare the impulse responses for different intertemporal smoothing baselines $\eta = 1$ and 0.5. To save space, the impulse responses are relegated in Appendix Figures 1 – 3. Appendix Figure 1 compares the impulse responses when the intratemporal smoothing value $\zeta = 1.2$ is more important than both intertemporal smoothing baselines. Appendix Figure 2 reports the impulse responses when the intratemporal smoothing value $\zeta = 1$ is as important as the intertemporal smoothing baseline $\eta = 1$, but is more important than the intertemporal smoothing baseline $\eta = 0.5$. Finally, Appendix Figure 3 is the impulse responses when the intratemporal smoothing value $\zeta = 0.4$ is less important than both intertemporal smoothing baselines.

As is clearly seen, in response to a positive housing demand shock, land prices increase more in the smaller intertemporal smoothing baseline $\eta = 0.5$ than the larger baseline $\eta = 1$ when the intratemporal smoothing values are larger at $\zeta = 1.2$ and 1, but land prices increase less in the smaller baseline $\eta = 0.5$ than the larger baseline $\eta = 1$ when the intratemporal smoothing value is smaller at $\zeta = 0.4$. Moreover, investment, labor hours and output increase more in the smaller baseline $\eta = 0.5$ than the larger baseline $\eta = 1$ when the intratemporal smoothing values are larger at $\zeta = 1.2$ and 1, but these macro variables increase less in the smaller baseline $\eta = 0.5$ than the larger baseline $\eta = 1$ when the intratemporal smoothing value is smaller at $\zeta = 0.4$. In particular, no matter whether the intratemporal smoothing value is $\zeta =$ 1.2, 1, or 0.4, aggregate consumption is always higher in the smaller baseline $\eta = 0.5$ than the larger baseline $\eta = 1$. However, when the intratemporal smoothing value is larger at $\zeta = 1.2$ and 1, aggregate consumption increases in the smaller baseline $\eta = 0.5$ but decreases in the larger baseline $\eta = 1$, while when the intratemporal smoothing value is smaller at $\zeta = 0.4$, aggregate consumption always decreases in both baselines $\eta = 0.5$ and 1. Thus, only when the intertemporal smoothing is $\eta = 0.5$ in the cases of larger intratemporal smoothing values $\zeta =$ 1.2 and 1, aggregate consumption increases and comoves with land prices, investment, labor hours, and output.

4.4 The role of η and ζ in the land price in response to the housing demand shock

The fluctuations of the land price are vital in the propagation of housing demand shocks via the collateral channel into impacts on the fluctuations in consumption, investment, and other variables. In the Appendix, we have derived the relationship between the land price $q_{l,t}$ and the housing demand shock φ_t, given as follows.

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t \Lambda_t(\eta,\zeta)}{\mu_{h,t}}, \tag{13}$$

where $\Lambda_t(\eta,\ \zeta) \equiv A_t \chi L_{h,t}^{\varphi_t(1-1/\zeta)-1} \left[(1-\chi)\left(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t}\right)^{1-1/\zeta} + \chi\left(L_{h,t}^{\varphi_t}\right)^{1-1/\zeta}\right]^{\frac{1-1/\eta}{1-1/\zeta}-1}$, and

$\mu_{h,t}$ is the Lagrange multiplier of the household's budget constraint in (11), which is the marginal utility of household's consumption in *t*, and $\varphi_t \Lambda_t(\eta, \zeta)$ is the marginal utility of household's land services in *t*.

In (13), the first term in the right-hand side, $\frac{\mu_{h,t+1}}{\mu_{h,t}}$ is the marginal rate of substitution (hereafter, MRS) of household's consumption between periods *t* and *t*+1. The second term in the right-hand side, $\frac{\varphi_t \Lambda_t(\eta, \zeta)}{\mu_{h,t}}$, is the MRS between household's consumption and land services within period *t*. When there is a positive housing demand shock (i.e., when φ_t increases), there are direct and indirect effects on the land price. There is a direct effect working through the term φ_t, which directly increases the volatility of the land price. There are indirect effects through affecting the MRS of household's consumption between periods *t* and *t*+1 and the MRS between household's consumption and land services within period *t*. These are where the intertemporal ES η and the intratemporal ES ζ exert effects.

To illustrate the role of η and ζ in the effect of the housing demand shock on the land price, let us simplify the household's utility by assuming no consumption habits and no growth factors, so $\gamma_h = 0$ and $A_t = \Gamma_t = 1$. Thus, the household's utility of consumption and land services in (8) reduces to

$$U\left(C_{h,t}, L_{h,t}\right) = \frac{1}{1-1/\eta}\left((1-\chi)\left(C_{h,t}\right)^{1-\frac{1}{\zeta}} + \chi\left(L_{h,t}^{\varphi_t}\right)^{1-\frac{1}{\zeta}}\right)^{\frac{1-1/\eta}{1-1/\zeta}}. \tag{14}$$

It serves to denote $\Delta_t(\varphi_t) \equiv (1-\chi)\left(C_{h,t}\right)^{1-\frac{1}{\zeta}} + \chi\left(L_{h,t}^{\varphi_t}\right)^{1-\frac{1}{\zeta}}$. Then, the marginal utility of household's consumption and the marginal utility of household's land services are, respectively, given by

$$\mu_{h,t} = (1-\chi)\left(C_{h,t}\right)^{-\frac{1}{\zeta}}(\Delta_t)^{\frac{1-1/\eta}{1-1/\zeta}-1},$$

$$\varphi_t \Lambda_t(\eta, \zeta) = \varphi_t \chi L_{h,t}^{\varphi_t(1-1/\zeta)-1}(\Delta_t)^{\frac{1-1/\eta}{1-1/\zeta}-1}.$$

In the special case of the Liu et al. (2013) model, $\zeta = \eta = 1$, and (14) reduces to the log separable form $U_t = (1-\chi)log\left(C_{h,t}\right) + \chi\varphi_t log L_{h,t}$. As a result, the marginal utility of household's consumption is $\mu_{h,t} = (1-\chi)(C_{h,t})^{-1}$ and the marginal utility of household's land services is $\varphi_t \Lambda_t(\eta, \zeta) = \varphi_t \chi (L_{h,t})^{-1}$, which is independent of the marginal utility of household's consumption. In this case, the relationship between the land price $q_{l,t}$ and the housing demand shock φ_t in (13) reduces to

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{C_{h,t}}{C_{h,t+1}} + \frac{\chi \varphi_t}{1-\chi} \frac{C_{h,t}}{L_{h,t}}. \tag{15}$$

Then, the housing demand shock exerts only the direct effect through $\frac{\chi\varphi_t}{1-\chi}$ on the land price.

By contrast, when any value of η and ζ deviates from unity, the effect changes. The relationship between the land price $q_{l,t}$ and housing demand shock φ_t in (13) now become

$$q_{l,t} = \beta E_t q_{l,t+1} \left(\frac{C_{h,t}}{C_{h,t+1}}\right)^{1/\zeta} \left(\frac{(1-\chi) C_{h,t}^{(1-1/\zeta)} + \chi L_{h,t}^{\varphi_t(1-1/\zeta)}}{(1-\chi) C_{h,t+1}^{(1-1/\zeta)} + \chi L_{h,t+1}^{\varphi_{t+1}(1-1/\zeta)}}\right)^{1-\frac{1-1/\eta}{1-1/\zeta}} + \frac{\chi\varphi_t}{1-\chi} \frac{C_{h,t}^{1/\zeta} L_{h,t}^{\varphi_t(1-1/\zeta)}}{L_{h,t}}. \tag{16}$$

Thus, the housing demand shock φ_t impacts the land price not only through the same direct effect $\frac{\chi\varphi_t}{1-\chi}$ as in Liu et al. (2013), but there are also other direct and indirect effect. Two cases are in order.

First, when $\eta = 1$ as in Liu et al. (2013) but $\zeta \neq 1$ different from Liu et al. (2013), there are indirect effects include those through $L_{h,t}^{\varphi_t(1-1/\zeta)}$ in the second term affecting the MRS between household's consumption and land services within period *t*. Thus, a different intratemporal consumption smoothing effect is at work.

Next, when $\eta < 1$ and $\zeta \neq 1$, there are other indirect effects acting through $L_{h,t}^{\varphi_t(1-1/\zeta)[1-(1-1/\eta)/(1-1/\zeta)]} \left(= L_{h,t}^{\varphi_t(\zeta-\eta)/(\eta\zeta)}\right)$ in the first term, which impact the MRS of household's consumption between periods *t* and *t*+1. Thus, a different intertemporal consumption smoothing effect is at work. In particular, consumption increases and comoves with land prices, investment, labor and output, only when $\eta < 1$ and $(\eta - \zeta) < 0$; that is, when the intertemporal consumption smoothing effect is sufficiently large and dominates the intratemporal consumption smoothing effect. In these cases, these stronger indirect effects cause the land price to increase more.[7] The resulting increase in the land price causes land

[7] This is seen by the impulse responses in Figure 2, wherein the land price increases the highest in the case $\zeta = 1.2$, followed by $\zeta = 1$, and then $\zeta = 0.4$.

services to be substituted away toward current consumption. Thus, consumption increases and comoves with land prices and business investment.

5. Relative importance of different structural shocks

We have compared the impulse responses of a positive housing demand shock in models when the intertemporal ES and the intratemporal ES differ from unity. This section compares relative importance of housing demand and other structural shocks in driving the impulse responses in the land price and other macroeconomic variables in models when the intertemporal ES and the intratemporal ES are different from unity. The relative importance of the shocks is performed through variance decomposition exercises.

5.1 Variance decomposition of different shocks in the baseline $\eta = 1$

Although the impulse responses have a comovement issue in the baseline $\eta = 1$, as this case was analyzed in Liu et al. (2013), this subsection starts with the baseline case $\eta = 1$. Tables 3a-3b report the variance decomposition of main aggregate variables across eight types of structural shocks at forecast horizons between the impact period (1Q) and six years after the shocks (24Q).

First, variance decompositions in Table 3a show that the housing demand shocks account for the lion's share of the fluctuations in the land price. For the fluctuations in the land price among different cases, the shares are the highest in the case $\zeta = 1.2$ accounting for 92%–94% in all forecast horizons from 1Q to 24Q, followed by the case $\zeta = 1$ in Liu et al. (2013), and then the case $\zeta = 0.4$. Propagated by the collateral constraints via increases in land prices, the housing demand shocks drive large fluctuations in business investment, output and labor hours. The shares of fluctuations in investment are the largest in the case $\zeta = 1.2$ accounting for 36%–40% in 1Q–24Q, followed by the case $\zeta = 1$, and then the case $\zeta = 0.4$. The shares of fluctuations in output are also the largest in the case $\zeta = 1.2$ accounting for 31%–36% in 1Q–24Q, which is followed by the case $\zeta = 1$ in 1Q–4Q but by the case $\zeta = 0.4$ in 8Q–24Q. Yet, for the shares of fluctuations in labor hours, the case $\zeta = 1$ is the largest accounting for 34%–44% in 1Q–24Q, followed by the case $\zeta = 1.2$ and then by the case $\zeta = 0.4$.

Next, the collateral shocks do not change land prices directly, but they impact the entrepreneur's borrowing capacity in a way similar to the housing demand shocks. The effects of collateral shocks are persistent. Table 3a indicates that collateral shocks account for about 5%–16% of fluctuations in investment, output, and hours for all forecast horizons for the case $\zeta = 1$ in Liu et al. (2013). For $\zeta = 1.2$, the shares of fluctuations in investment and output are a little bit larger than those in Liu et al. (2013). For $\zeta = 0.4$, the share of fluctuations in investment, output, and labor hours are a little bit smaller than those of the case $\zeta = 1$ in Liu et al. (2013).

Moreover, the labor supply shocks also drive large fluctuations in output and labor hours. Yet, the shares in both cases $\zeta = 1.2$ and 0.4 are larger than those of $\zeta = 1$ in Liu et al. (2013). In particular, in the case $\zeta = 0.4$, the labor supply shocks account for large shares of the fluctuations in the land price and investment, which are different from the case $\zeta = 1$ in Liu et al. (2013).

Furthermore, the patience shocks can drive large fluctuations in investment, output and labor hours. Yet, no matter whether it is the case $\zeta = 1.2$ or 0.4, the shares of fluctuations in land prices, investment, output, and labor hours are larger than those of the case $\zeta = 1$ in Liu et al. (2013).

Finally, as in Liu et al. (2013), permanent and transitory shocks to the total factor productivity (henceforth TFP) in Table 3b contribute little to fluctuations in the land price, investment, and labor hours in all cases $\zeta = 1.2$, 1 and 0.4, but permanent and transitory shocks to TFP account for a large fluctuation in output in the case $\zeta = 1$, but not in cases $\zeta = 1.2$ and 0.4. As in Liu et al. (2013), permanent and transitory shocks to investment-specific technology (henceforth IST) contribute little to land price, investment, output and labor hours.

5.2 Variance decomposition of different shocks in the baseline $\eta = 0.5$

Now, we compare relative importance of structural shocks in driving the impulse responses in the land price and other macroeconomic variables when the baseline is $\eta = 0.5$. Tables 4a–4b report variance decompositions of main aggregate variables across eight types of structural shocks.

First, Table 4a indicates that the housing demand shock accounts for larger shares of the fluctuations in the land price in the case $\zeta = 1$ than the corresponding case in Table 3a under the baseline η =1. Moreover, propagated by the collateral constraints via large fluctuations in land prices, fluctuations in investment, output and labor hours are also larger in the case $\zeta = 1$, as compared to the corresponding case $\zeta = 1$ in Table 3a. For the cases ζ =1.2 and 0.4, Table 4a also indicates that shocks to the housing demand still account for large shares of the fluctuations in the land price, but their shares are slightly smaller than those in the corresponding cases in Table 3a. Moreover, propagated by the collateral constraints via large changes in land prices, the housing demand shocks also drive large fluctuations in investment, output and hours.

It is interesting to compare with Gong et al. (2017), which extended Liu et al. (2013) to consider the intertemporal ES of $\eta = 0.5$. These authors found that the housing demand shocks drive large fluctuations in land prices, but the shares of the fluctuations in land prices, investment, output and labor hours under $\eta = 0.5$ are all smaller than those under $\eta = 1$ in Liu et al. (2013). By contrast, in our Table 4a, which is under $\eta = 0.5$, in response to a positive housing demand shock, the shares of the fluctuations in land prices, investment, output and labor hours in the case $\zeta = 1$ are larger than those under $\eta = 1$ in Liu et al. (2013). The difference arises, because Gong et al. (2017) allowed for a finite Frisch ELS, different from an infinite Frisch ELS in both Liu et al. (2013) and our model. A finite Frisch ELS by itself can reduce the amplification effect of the credit constraint mechanism triggered by a positive housing demand shock on key macroeconomic variables.

Next, directly impacting the entrepreneur's borrowing capacity, collateral shocks are persistent and drive large fluctuations in investment, output, and labor hours, like those in Table 3a. Thus, collateral shocks drive similar fractions of fluctuations in investment, output and labor hours, as in Table 3a.

Moreover, shocks to the labor supply drive slightly larger fluctuations in the land price, and thus, slightly larger fluctuations in investment in Table 4a than in Table 3a. The shares of the fluctuations in labor hours in Table 4a are also larger than those in Table 3a. Yet, the fluctuations in output in Table 4a are smaller than those in Table 3a. Furthermore, patience shocks also can drive similar, though slightly larger, fluctuations in the land price, investment and output. The shares of fluctuations in labor hours are smaller than those in Table 3a.

Finally, in Table 4b, shocks to the TFP and shocks to IST, both permanent and transitory, contribute little to fluctuations in the land price, investment and labor hours, like Table 3b.

5.3 Comparing different baselines values of η and different cases of ζ

With a given baseline intertemporal ES η, in order to understand whether a model with an intratemporal ES ζ different from unity is favored by the data, we report the marginal data density (henceforth MDD). Given data set, the MDD measures how likely the model is supported by the data.[8] The MDD is the most comprehensive measure of fit. As in Liu et al. (2016), we estimate the MDD using three different methods based on different theoretical foundations. SWZ is the method developed by Sims et al. (2008), Mueller is the Mueller method described in Liu et al. (2011), and Bridge is the bridge-sampling method proposed by Meng and Wong (1996).

First, in the baseline η = 1, Table 5a reports the MDD values for different values of the intratemporal ESs, ζ = 0.4, 1 and 1.2. Thus, we compare the cases ζ = 0.4 and 1.2 with the counterpart case ζ= 1 in Liu et al. (2013). An inspection of Table 5a reveals that the model with ζ=1.2 has the highest MDD values and the model with ζ=0.4 has the lowest MDD values for all three different estimation methods. Thus, with the baseline intertemporal ES given at η = 1, the data is in favor of the model with ζ=1.2.

Next, in the other baseline η = 0.5, we also report the MDD for different values of the intratemporal ES, ζ = 0.4, 1 and 1.2. Using the same methods as in Table 5a, Table 5b reports the MDD values for cases ζ = 0.4, 1 and 1.2. An assessment of Table 5b suggests that, under the baseline η= 0.5, the model with ζ=1.2 stays the highest MDD values and the model with ζ =0.4 has the lowest MDD values for all different estimation methods. Thus, with the baseline intertemporal ES given at η = 0.5, the data is in favor of the model with ζ = 1.2.

It is interesting to see whether the data is in favor of the baseline η = 1 or the baseline η = 0.5. To this end, we compare the MDD values between Tables 5a and 5b. The result suggests that the model of η = 0.5 and ζ = 1.2 has the largest MDD values for three MDD measures among all six models. Thus, the model with the intertemporal ES less than unity and the intratemporal ES larger than unity is favored by the data. That is, the model is in favor of a household's utility with a complementary relationship for consumption bundles across periods

[8] The DSGE-VAR approach, as proposed by Del Negro and Schorfheide (2004) and Del Negro et al. (2007), requires the number of shocks equal the number of observed variables. Yet, under the framework of Liu et al. (2013), there are eight shocks and six observed variables, which makes the DSGE-VAR method infeasible. Thus, we report the value of MDD in the same way as in Gong et al. (2017).

but a substitutable relationship between consumption and land services within a period.

5.4 Impulse responses and variance decomposition for estimated values of η and ζ

The impulse responses and variance decomposition obtained above are under the values of the two parameters η and ζ set from the outside. The amplification effect through the borrowing constraint channel in our model may be sensitive to the values of these two key structural parameters. To put our results on a robust groundwork, we now have these two key parameters structurally estimated within the current model and let the data and the model decide what the estimates of these two elasticities of substitution should be.

Specifically, we estimate the values for the intertemporal ES and the intratemporal ES using Bayesian techniques. The prior distributions of all parameters except η and ζ are the same as those in Liu et al. (2013). Given that the intertemporal ES is positive and less than one ($0<\eta<1$), we set the prior distribution of η to be a standard uniform distribution $U(a, b)$ with $(a, b)=(0, 1)$, which are the minimum and maximum values. Moreover, given that the intratemporal ES may be less than or larger than one ($\zeta<1$ or $\zeta>1$), we set the prior distribution of ζ to be Gamma(a, b) with $(a, b)=(1.00, 0.50)$, which are the shape parameter value of $a=1.00$ and the inverse scale parameter value of $b=1/2$.

The prior distribution and the estimated posterior modes are in Tables 6a and 6b. Table 6a is the estimated posterior modes of the structural parameters, wherein the estimated value $\eta=0.4760$ is close to the value $\eta=0.5$ set from the outside in previous sections, whereas the estimated value $\zeta=2.7722$ is larger than the value $\zeta=1.2$ set from the outside in previous analyses. With these two structurally estimated values of η and ζ, we estimate the posterior modes of the shock parameters and report in Table 6b, which indicate that the estimated posterior modes of all parameters, except ϕ, $\bar{\varphi}$, ρ_{vq} and σ_a, are close to those in the model with the largest MDD values and parameter values $\eta = 0.5$ and $\zeta = 1.2$ set from the outside.

Under these two structurally estimated parameter values $\eta=0.4760$ and $\zeta=2.7722$, Figure 3 illustrates the impulse responses of the housing demand shock, along with the impulse responses under the values $\eta=0.5$ and $\zeta=1.2$ set from the outside. Figure 3 suggests that, under the model with the two estimated parameter values $\eta=0.4760$ and $\zeta=2.7722$, aggregate consumption increases, and comoves with land prices, business investment, labor hours and output, like those in the model under the values $\eta=0.5$ and $\zeta=1.2$ set from the outside.

Comparing the model of η=0.4760 and ζ=2.7722 with the model of η =0.5 and ζ=1.2, the willingness to substitute consumption for land in a period in the model of ζ=2.7722 is larger than the willingness in the model of ζ =1.2, but the willingness to substitute consumption across periods in the model of η=0.4760 is smaller than the willingness in the model of η=0.5. Thus, consumption increases more but the land price increases less in the model of η=0.4760 and ζ=2.7722 than the model of η =0.5 and ζ=1.2 in Figure 3. As the entrepreneurs increase investment a little bit more and also labor hours more in the model of η=0.4760 and ζ=2.7722 than the model of η =0.5 and ζ=1.2, output also increases more in the model of η=0.4760 and ζ=2.7722.

Table 7 reports the variance decomposition of the main aggregate variables across eight types of structural shocks at forecast horizons from one to twenty-four quarters under the model with the estimated values of η=0.4760 and ζ=2.7722. Like the variance decomposition in Tables 4a-4b for the model with η =0.5 and ζ=1.2 set from the outside, the variance decomposition in Table 7 indicates that, under these two structurally estimated parameter values, the housing demand shock is still the main driving force for the fluctuation of the land price, investment, output and working hours.

Finally, we wonder whether or not the data is more in favor of the model with these two structurally estimated parameter values η=0.4760 and ζ=2.7722 than the model above with parameter values η = 0.5 and ζ = 1.2 are set from the outside that gives the largest MDD, as reported in Table 5b. Table 8 reports three MDD values for the estimated values of η and ζ. A comparison of Table 8 with Table 5b suggests that the data is in support of the model with these two estimated values η=0.4760 and ζ=2.7722. This result further confirms our previous analysis that the data is in favor of the model with a household's utility with a complementary relationship for consumption bundles across periods but a substitutable relationship between consumption and land services within a period.

6. Concluding remarks

Liu et al. (2013) have recently introduced land as a collateral asset in firms' borrowing constraints and found that a positive shock to housing demand generates a mechanism that amplifies and propagates the shock through the joint dynamics of land prices and business investment. Their estimation showed that the housing demand shock accounts for about 90% of the fluctuations in land prices, along with large fluctuations in investment, output, and labor hours. Yet, their impulse responses have a comovement puzzle, as consumption decreases, which is different from the estimated evidence from a BVAR model, wherein consumption increases and comoves with land prices, investment, and hours. The comovement puzzle emerges from their household's utility that is log-separable in consumption and land services. Their utility implies that the intertemporal ES and the intratemporal ES both are unity. However, the existing estimates indicate that the intertemporal ES is statistically and significantly less than unity, and the intratemporal ES is greater than intertemporal ES.

Our paper introduces the CES utility between consumption and land services into Liu et al. (2013), which reduces to the log-separable utility if the intertemporal ES and the intratemporal ES are unity. We investigate whether, following a positive housing demand shock, the comovement puzzle can be resolved and the striking results of Liu et al. (2013) are changed if the intertemporal ES and the intratemporal ES deviate from unity. We find that, as in Liu et al. (2013), a positive housing demand shock leads to persistent increases not only in land prices but also in business investment, as land prices trigger a financial multiplier that interacts land prices with business investment in propagating the shock, and the housing demand shock accounts for the lion's share of the fluctuations in the land price. However, as the intertemporal ES and the intratemporal ES are different from unity, there are three different results.

First, consumption increases and comoves with land prices and business investment, when the intertemporal ES is less than unity and smaller than the intratemporal ES; that is, when the intertemporal consumption smoothing effect is sufficiently large and dominates the intratemporal consumption smoothing effect. The condition generates stronger indirect effects, which increase the land price by more, thus causing land services to be substituted away toward current consumption. As a result, consumption increases and comoves with land prices and business investment.

Next, relative to other shocks, the housing demand shock accounts for the lion's share of the fluctuations in the land price, but the fractions of the fluctuation in macroeconomic variables accounted for by the housing demand shock are different from those in Liu et al. (2013), when the intertemporal ES and the intratemporal ES are different from unity.

Finally, using three measures of the MDD, we find that the model with the intertemporal ES less than unity and the intratemporal ES larger than unity is preferred. Thus, the data lends support to the model with the household's preference featuring a complementary relationship for consumption bundles across periods and a substitutable relationship for consumption and land services within a period.

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Table 1a: Prior distributions and posterior modes of the structural parameters

Parameter	Prior				Posterior mode		
	Distribution	description	a	b	ζ=1	ζ=0.4	ζ=1.2
γ_h	Beta (a,b)	Household's habit persistence	1.00	2.00	0.4976	0.5790	0.4926
γ_e	Beta (a,b)	Entrepreneur's habit persistence	1.00	2.00	0.6584	0.6613	0.6601
Ω	Gamma(a,b)	Capital adjustment cost parameter	1.00	0.50	0.1753	0.2014	0.2083
$100(g_\gamma - 1)$	Gamma(a,b)	Steady state growth rate of TFP	1.86	3.01	0.4221	0.4177	0.4137
$100(\lambda_q - 1)$	Gamma(a,b)	Steady state growth rate of IST	1.86	3.01	1.2126	1.1704	1.2064
β	Calibrated	Discount factor			0.9855	0.9857	0.9858
$\bar{\lambda}_a$	Calibrated	Steady state growth rate of preference shock			0.0091	0.0098	0.0085
ϕ	Calibrated	Share on land input			0.0695	0.0696	0.0696
δ	Calibrated	Depreciation rate			0.0368	0.0372	0.0369
$\bar{\varphi}$	Calculated	Steady state of housing demand shock			0.0457	0.0461	0.0463

Note: 1. In the table, we set η =1.
2. Case ζ=1 reduces to Liu et al. (2013).

Table 1b: Prior distributions and posterior modes of the shock parameters

Para-meter	Prior				Posterior mode		
	Distribution	description	a	b	ζ=1	ζ=0.4	ζ=1.2
ρ_a	Beta (a,b)	Intertemporal preference shock	1.00	2.00	0.9055	0.8948	0.9015
ρ_z	Beta (a,b)	Permanent neutral technology shock	1.00	2.00	0.4263	0.4193	0.4210
ρ_{vz}	Beta (a,b)	Transitory neutral technology shock	1.00	2.00	0.0095	0.0072	0.0039
ρ_q	Beta (a,b)	Permanent shock to IST change	1.00	2.00	0.5620	0.5745	0.5637
ρ_{vq}	Beta (a,b)	Transitory shock to IST change	1.00	2.00	0.2949	0.2999	0.2954
ρ_φ	Beta (a,b)	Housing demand shock	1.00	2.00	0.9997	0.9998	0.9998
ρ_ψ	Beta (a,b)	Labor supply shock	1.00	2.00	0.9829	0.9526	0.9922
ρ_ς	Beta (a,b)	Collateral shock	1.00	2.00	0.9804	0.9785	0.9811
σ_a	Inv-Gamma(a,b)	SD, Intertemporal preference shock	0.3261	1.45E-04	0.1013	0.1033	0.0865
σ_z	Inv-Gamma(a,b)	SD, Permanent neutral tech shock	0.3261	1.45E-04	0.0042	0.0096	0.0079
σ_{vz}	Inv-Gamma(a,b)	SD, Transitory neutral tech shock	0.3261	1.45E-04	0.0037	0.0126	0.0097
σ_q	Inv-Gamma(a,b)	SD, Permanent shock to IST change	0.3261	1.45E-04	0.0042	0.0038	0.0039
σ_{vq}	Inv-Gamma(a,b)	SD, Transitory shock to IST change	0.3261	1.45E-04	0.0029	0.0030	0.0028
σ_φ	Inv-Gamma(a,b)	SD, Housing demand shock	0.3261	1.45E-04	0.0462	0.0470	0.0480
σ_ψ	Inv-Gamma(a,b)	SD, Labor supply shock	0.3261	1.45E-04	0.0073	0.0078	0.0097
σ_ς	Inv-Gamma(a,b)	SD, Collateral shock	0.3261	1.45E-04	0.0112	0.0127	0.0121

Note: Same as Table 1a.

Table 2a: Prior distributions and posterior modes of the structural parameters

Para-meter	Prior distribution	Description	a	b	Posterior mode ζ=1	ζ=0.4	ζ=1.2
γ_h	Beta (a,b)	Household's habit persistence	1.00	2.00	0.5171	0.5444	0.6304
γ_e	Beta (a,b)	Entrepreneur's habit persistence	1.00	2.00	0.6652	0.8264	0.6671
Ω	Gamma(a,b)	Capital adjustment cost parameter	1.00	0.50	0.1832	0.2123	0.1997
$100(g_\gamma - 1)$	Gamma(a,b)	Steady state growth rate of TFP	1.86	3.01	0.4183	0.4199	0.3969
$100(\lambda_q - 1)$	Gamma(a,b)	Steady state growth rate of IST	1.86	3.01	1.2066	1.2109	1.1537
β	Calibrated	Discount factor			0.9856	0.9856	0.9862
$\bar{\lambda}_a$	Calibrated	Steady state growth rate of preference shock			0.0087	0.0088	0.0079
ϕ	Calibrated	Share on land input			0.0695	0.0695	0.0696
δ	Calibrated	Depreciation rate			0.0369	0.0368	0.0376
$\bar{\varphi}$	Calculated	Steady state of housing demand shock			0.0460	0.0459	0.0478

Note: In the table, we set η = 0.5.

Table 2b: Prior distributions and posterior modes of the shock parameters

Para-meter	Prior distribution	Description	a	b	Posterior mode ζ=1	ζ=0.4	ζ=1.2
ρ_a	Beta (a,b)	Intertemporal preference shock	1.00	2.00	0.9071	0.9001	0.9076
ρ_z	Beta (a,b)	Permanent neutral technology shock	1.00	2.00	0.4315	0.4246	0.4291
ρ_{vz}	Beta (a,b)	Transitory neutral technology shock	1.00	2.00	0.0076	0.0018	0.0016
ρ_q	Beta (a,b)	Permanent shock to IST change	1.00	2.00	0.5630	0.5574	0.6017
ρ_{vq}	Beta (a,b)	Transitory shock to IST change	1.00	2.00	0.2951	0.2962	0.3078
ρ_φ	Beta (a,b)	Housing demand shock	1.00	2.00	0.9997	0.9999	0.9997
ρ_ψ	Beta (a,b)	Labor supply shock	1.00	2.00	0.9992	0.9899	0.9985
ρ_ς	Beta (a,b)	Collateral shock	1.00	2.00	0.9801	0.9810	0.9831
σ_a	Inv-Gamma(a,b)	SD, intertemporal preference shock	0.326	1.45E-04	0.0901	0.0801	0.0085
σ_z	Inv-Gamma(a,b)	SD, permanent neutral tech shock	0.326	1.45E-04	0.0079	0.0119	0.0071
σ_{vz}	Inv-Gamma(a,b)	SD, transitory neutral tech shock	0.326	1.45E-04	0.0092	0.0118	0.0091
σ_q	Inv-Gamma(a,b)	SD, permanent shock to IST change	0.326	1.45E-04	0.0039	0.0040	0.0037
σ_{vq}	Inv-Gamma(a,b)	SD, transitory shock to IST change	0.326	1.45E-04	0.0028	0.0029	0.0029
σ_φ	Inv-Gamma(a,b)	SD, housing demand shock	0.326	1.45E-04	0.0502	0.0381	0.0493
σ_ψ	Inv-Gamma(a,b)	SD, labor supply shock	0.326	1.45E-04	0.0159	0.0152	0.0125
σ_ς	Inv-Gamma(a,b)	SD, collateral shock	0.326	1.45E-04	0.0121	0.0126	0.0122

Note: Same as Table 2a.

Table 3a: Variance decomposition of aggregate quantities

	Land price			Investment			Output			Hours		
	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2
Patience shocks												
1Q	4.09	5.09	4.34	19.37	20.6	19.32	12.28	15.43	12.97	12.46	17.26	14.62
4Q	3.30	4.18	3.52	18.80	20.05	18.93	11.22	14.38	12.25	11.88	16.23	13.88
8Q	2.91	3.75	3.13	17.23	19.91	18.76	9.68	13.79	11.97	10.72	13.99	12.40
16Q	2.29	3.05	2.52	14.91	19.31	18.27	7.43	12.21	11.15	9.29	11.21	10.59
24Q	1.77	2.43	1.99	13.56	18.92	17.99	5.97	10.87	10.46	8.68	9.97	9.78
Housing demand shocks												
1Q	89.99	86.73	92.80	35.46	33.16	36.66	27.82	25.21	31.89	44.87	28.2	35.94
4Q	90.74	86.78	92.94	41.19	38.08	42.18	31.80	29.55	36.65	44.94	27.71	36.55
8Q	90.28	86.33	92.72	38.71	37.85	42.41	28.32	28.35	36.48	42.50	24.07	34.38
16Q	89.58	86.82	93.39	33.70	36.34	41.33	21.82	24.20	33.65	37.54	19.28	29.94
24Q	89.27	87.66	94.29	30.67	35.75	40.75	17.37	21.16	31.11	34.75	17.36	27.55
Labor supply shocks												
1Q	2.55	7.40	2.18	12.06	16.96	12.46	21.85	31.47	24.11	20.20	35.20	27.18
4Q	2.25	6.86	1.86	12.02	17.19	12.22	21.13	32.49	23.43	24.08	41.62	31.11
8Q	2.41	7.17	2.01	12.56	18.99	13.59	22.22	38.28	27.26	29.75	49.02	36.85
16Q	2.68	7.84	2.27	13.00	21.39	15.64	23.85	47.91	34.32	37.68	56.79	44.84
24Q	2.72	8.15	2.31	12.63	22.19	16.38	23.87	54.22	39.31	41.45	59.98	48.43
Collateral shocks												
1Q	0.00	0.04	0.01	12.33	12.28	13.12	9.17	8.10	10.54	13.82	9.06	11.88
4Q	0.11	0.29	0.10	16.08	15.82	16.93	12.12	11.32	14.17	13.09	8.28	11.54
8Q	0.25	0.44	0.23	14.65	15.23	16.43	10.32	10.33	13.51	11.56	6.53	10.01
16Q	0.35	0.49	0.35	12.42	14.35	15.54	7.38	8.21	11.50	9.99	5.85	8.48
24Q	0.29	0.39	0.29	11.51	14.45	15.62	5.74	7.17	10.35	9.83	6.28	8.46

Note: 1. η =1.
2. Case ζ=1 reduces to Liu et al. (2013).

Table 3b: Variance decomposition of aggregate quantities

	Land price			Investment			Output			Hours		
	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2
Permanent shocks to TFP												
1Q	1.97	0.10	2.18	1.13	0.05	0.00	6.93	5.08	4.53	0.43	0.89	0.56
4Q	3.19	0.44	1.86	5.64	1.03	1.48	17.14	1.00	0.82	0.61	0.98	1.00
8Q	3.84	0.45	2.01	9.19	1.98	2.53	25.20	0.59	0.48	1.27	2.08	1.87
16Q	4.88	0.27	2.27	12.71	2.42	2.94	35.70	0.44	0.36	1.49	2.22	1.99
24Q	5.68	0.19	2.31	14.41	2.42	2.94	42.82	0.38	0.32	1.42	1.98	1.86
Transitory shocks to TFP												
1Q	1.25	0.06	0.13	14.30	7.08	7.94	16.06	0.02	0.01	1.48	1.32	1.36
4Q	0.34	0.07	0.46	4.95	3.92	4.27	4.73	2.98	3.20	2.69	2.66	2.97
8Q	0.22	0.13	0.43	3.70	3.28	3.53	3.19	2.55	2.88	2.25	2.00	2.40
16Q	0.17	0.14	0.25	3.11	3.08	3.34	2.29	2.18	2.64	1.95	1.59	2.04
24Q	0.13	0.12	0.17	2.83	3.02	3.29	1.84	1.93	2.49	1.81	1.42	1.88
Permanent shocks to IST												
1Q	0.01	0.27	0.15	3.01	9.20	9.93	5.34	14.59	15.91	6.40	6.97	7.76
4Q	0.06	1.26	0.06	0.88	3.78	3.84	1.75	8.25	9.45	2.61	2.22	2.75
8Q	0.08	1.67	0.09	3.63	2.60	2.61	0.99	6.07	7.38	1.84	2.04	1.91
16Q	0.05	1.35	0.10	9.86	2.91	2.77	1.47	4.79	6.34	1.95	2.82	1.94
24Q	0.13	1.03	0.09	14.13	3.06	2.85	2.35	4.20	5.92	1.96	2.79	1.89
Transitory shocks to IST												
1Q	0.03	0.39	0.12	2.34	0.67	0.55	0.57	0.00	0.03	0.35	1.23	0.70
4Q	0.01	0.16	0.98	0.44	0.27	0.15	0.11	0.03	0.02	0.11	0.55	0.20
8Q	0.01	0.08	1.35	0.32	0.23	0.16	0.07	0.04	0.04	0.12	0.41	0.18
16Q	0.00	0.04	1.09	0.29	0.28	0.17	0.06	0.05	0.04	0.11	0.37	0.17
24Q	0.00	0.03	0.83	0.26	0.28	0.17	0.05	0.05	0.04	0.11	0.32	0.16

Note: 1. η =1.

2. Case ζ=1 reduces to Liu et al. (2013).

Table 4a: Variance decomposition of aggregate quantities

	Land price			Investment			Output			Hours		
	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2
Patience shocks												
1Q	4.49	6.37	4.38	14.91	22.28	19.85	9.43	17.03	13.95	9.79	18.09	15.26
4Q	3.67	5.42	3.56	14.89	21.42	19.31	8.67	15.45	12.75	9.04	16.59	14.12
8Q	3.29	4.98	3.19	14.72	21.01	18.92	8.16	14.33	12.03	7.91	13.79	12.00
16Q	2.67	4.17	2.57	14.18	20.20	18.18	7.30	12.18	10.53	6.77	11.05	9.73
24Q	2.11	3.39	2.03	13.85	19.67	17.76	6.69	10.59	9.39	6.32	10.13	8.78
Housing demand shocks												
1Q	90.57	78.91	91.40	42.66	30.00	36.39	44.12	23.68	32.15	45.78	25.16	35.17
4Q	90.59	79.23	91.44	47.00	34.82	42.11	45.58	26.81	36.21	46.02	24.13	35.19
8Q	90.04	78.54	90.98	47.01	34.79	41.93	45.02	25.43	35.35	44.23	20.38	31.66
16Q	90.44	78.45	91.46	45.83	33.44	40.33	42.23	21.56	31.21	40.33	16.35	26.02
24Q	91.11	78.82	92.20	44.81	32.57	39.44	39.34	18.63	27.69	37.40	14.99	23.13
Labor supply shocks												
1Q	4.19	13.97	3.48	13.90	21.20	13.61	20.38	36.26	25.92	21.15	38.52	28.35
4Q	3.83	13.44	3.14	14.10	21.49	13.64	20.98	37.90	26.46	24.33	44.82	33.73
8Q	4.15	14.01	3.41	15.61	23.17	15.28	24.46	43.81	31.75	28.70	51.02	40.40
16Q	7.82	15.29	3.96	18.23	25.92	17.92	31.06	53.31	41.20	35.67	57.00	49.13
24Q	5.21	16.18	4.24	19.77	27.28	19.21	36.43	59.65	47.92	40.09	59.52	53.45
Collateral shocks												
1Q	0.01	0.00	0.01	13.38	11.32	12.28	15.07	9.09	10.52	15.63	9.66	11.51
4Q	0.10	0.16	0.09	15.92	14.53	16.10	16.55	11.68	13.48	15.19	8.77	10.74
8Q	0.24	0.31	0.23	15.17	13.73	15.47	15.64	10.53	12.44	13.80	6.89	8.80
16Q	0.34	0.40	0.34	14.05	12.58	14.48	13.51	8.28	10.12	11.91	6.04	7.39
24Q	0.28	0.32	0.28	13.87	12.54	14.46	12.07	7.09	8.84	11.16	6.33	7.43

Note: 1. η = 0.5

Table 4b: Variance decomposition of aggregate quantities

	Land price			Investment			Output			Hours		
	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2	ζ=1	ζ=0.4	ζ=1.2
Permanent shocks to TFP												
1Q	0.07	0.01	0.08	0.00	0.01	0.14	1.20	3.33	4.28	0.05	0.46	0.74
4Q	0.50	0.24	0.50	1.33	1.28	1.16	0.23	0.59	0.84	0.96	1.40	1.18
8Q	0.55	0.25	0.53	2.27	2.34	2.41	0.27	0.52	0.54	1.76	2.94	2.61
16Q	0.34	0.13	0.33	2.71	2.88	2.96	0.28	0.48	0.43	1.92	3.08	2.87
24Q	0.24	0.10	0.23	2.75	2.95	2.95	0.27	0.42	0.38	1.82	2.72	2.60
Transitory shocks to TFP												
1Q	0.22	0.11	0.21	7.23	6.93	8.38	0.60	0.09	0.04	2.65	2.23	2.21
4Q	0.08	0.08	0.07	3.65	3.40	4.28	2.73	2.56	3.24	2.77	2.40	3.09
8Q	0.10	0.14	0.10	3.03	2.80	3.52	2.44	2.09	2.79	2.27	1.70	2.34
16Q	0.11	0.16	0.11	2.83	2.60	3.29	2.22	1.72	2.42	1.97	1.37	1.88
24Q	0.10	0.14	0.10	2.76	2.54	3.21	2.07	1.50	2.17	1.82	1.27	1.68
Permanent shocks to IST												
1Q	0.13	0.18	0.12	7.54	7.55	8.77	9.19	10.50	13.14	4.62	4.40	5.86
4Q	1.14	1.22	1.10	3.01	2.82	3.25	5.24	4.97	7.00	1.58	1.26	1.70
8Q	1.58	1.69	1.53	2.04	1.92	2.32	3.94	3.23	5.04	1.18	2.79	1.95
16Q	1.25	1.36	1.21	2.01	2.08	2.67	3.32	2.37	4.02	1.27	4.66	2.77
24Q	0.94	1.02	0.91	2.03	2.17	2.78	3.06	2.03	3.57	1.25	4.67	2.74
Transitory shocks to IST												
1Q	0.32	0.44	0.31	0.38	0.71	0.56	0.01	0.02	0.00	0.34	1.47	0.89
4Q	0.09	0.20	0.09	0.10	0.24	0.15	0.03	0.04	0.02	0.11	0.63	0.25
8Q	0.04	0.09	0.04	0.15	0.23	0.17	0.07	0.07	0.05	0.15	0.48	0.23
16Q	0.02	0.04	0.02	0.16	0.29	0.19	0.08	0.09	0.06	0.15	0.43	0.21
24Q	0.01	0.03	0.01	0.16	0.29	0.18	0.08	0.08	0.05	0.14	0.38	0.18

Note: 1. η = 0.5.

Table 5a: Measures of model fit for different ζ values under η =1

Fit measure (log value)	$\zeta = 0.4$	$\zeta = 1$	$\zeta = 1.2$
MDD (SWZ)	2287.94	2454.57	2469.16
MDD (Mueller)	2285.37	2452.51	2467.31
MDD (Bridge)	2285.55	2452.28	2467.38

Table 5b: Measures of model fit for different ζ values under η = 0.5

Fit measure (log value)	$\zeta = 0.4$	$\zeta = 1$	$\zeta = 1.2$
MDD (SWZ)	2389.92	2517.82	2525.74
MDD (Mueller)	2388.56	2516.01	2523.41
MDD (Bridge)	2388.95	2515.99	2523.41

Table 6a: Prior distributions and posterior modes of the structural parameters

Parameter	Prior				Posterior mode
	distribution	Description	a	b	η=0.4760 ζ=2.7722
γ_h	Beta (a,b)	Household's habit persistence	1.00	2	0.3943
γ_e	Beta (a,b)	Entrepreneur's habit persistence	1.00	2	0.7852
Ω	Gamma(a,b)	Capital adjustment cost parameter	1.00	0.5	0.3354
$100(g_\gamma - 1)$	Gamma(a,b)	Steady state growth rate of TFP	1.86	3.01	0.4718
$100(\lambda_q - 1)$	Gamma(a,b)	Steady state growth rate of IST	1.86	3.01	1.1101
η	Uniform(a,b)	Intertemporal ES	0	1	0.4760
ζ	Gamma(a,b)	Intratemporal ES	1.00	0.5	2.7722
β	Calibrated	Discount factor			0.9842
$\bar{\lambda}_a$	Calibrated	Steady state growth rate of preference shock			0.0108
ϕ	Calibrated	Share on land input			0.7496
δ	Calibrated	Depreciation rate			0.0373
$\bar{\varphi}$	Calculated	Steady state of housing demand shock			0.4514

Table 6b: Prior distributions and posterior modes of the shock parameters

Para-meter	Prior distribution	Description	a	b	Posterior mode η=0.4760 ζ=2.7722
ρ_a	Beta (a,b)	Intertemporal preference shock	1.00	2.00	0.8889
ρ_z	Beta (a,b)	Permanent neutral technology shock	1.00	2.00	0.6482
ρ_{vz}	Beta (a,b)	Transitory neutral technology shock	1.00	2.00	0.0010
ρ_q	Beta (a,b)	Permanent shock to IST change	1.00	2.00	0.6738
ρ_{vq}	Beta (a,b)	Transitory shock to IST change	1.00	2.00	0.4665
ρ_φ	Beta (a,b)	Housing demand shock	1.00	2.00	0.9999
ρ_ψ	Beta (a,b)	Labor supply shock	1.00	2.00	0.9884
ρ_ς	Beta (a,b)	Collateral shock	1.00	2.00	0.9752
σ_a	Inv-Gamma(a,b)	SD on intertemporal preference shock	0.326	1.45E-04	0.1648
σ_z	Inv-Gamma(a,b)	SD on permanent neutral tech shock	0.326	1.45E-04	0.0032
σ_{vz}	Inv-Gamma(a,b)	SD on transitory neutral tech shock	0.326	1.45E-04	0.0070
σ_q	Inv-Gamma(a,b)	SD on permanent shock to IST change	0.326	1.45E-04	0.0029
σ_{vq}	Inv-Gamma(a,b)	SD on transitory shock to IST change	0.326	1.45E-04	0.0038
σ_φ	Inv-Gamma(a,b)	SD on housing demand shock	0.326	1.45E-04	0.0525
σ_ψ	Inv-Gamma(a,b)	SD on labor supply shock	0.326	1.45E-04	0.0082
σ_ς	Inv-Gamma(a,b)	SD on collateral shock	0.326	1.45E-04	0.0122

Note: Same as Table 2a.

Table 7: Variance decomposition of aggregate quantities

Shocks	Patience	Housing demand	Labor supply	Collateral	TFP permanent	TFP transitory	IST permanent	IST transitory
Horizon				Land price				
1Q	3.58	94.03	1.62	0.08	0.13	0.20	0.08	0.29
4Q	2.88	94.21	1.36	0.04	0.47	0.06	0.89	0.08
8Q	2.55	93.99	1.50	0.13	0.43	0.08	1.28	0.04
16Q	2.03	94.57	1.75	0.25	0.24	0.10	1.04	0.02
24Q	1.59	95.34	1.80	0.21	0.17	0.09	0.79	0.01
				Investment				
1Q	15.79	41.40	10.69	13.59	0.00	8.14	9.86	0.53
4Q	15.68	46.79	10.69	16.81	1.44	4.47	3.99	0.15
8Q	15.56	47.21	12.03	16.15	2.44	3.74	2.72	0.16
16Q	15.15	46.26	13.99	15.25	2.83	3.55	2.80	0.18
24Q	14.93	45.60	14.76	15.36	2.82	3.49	2.85	0.18
				Output				
1Q	9.84	40.45	18.86	13.84	2.79	0.06	14.12	0.04
4Q	9.24	43.72	18.25	16.26	0.49	3.31	8.72	0.02
8Q	9.00	43.94	21.26	15.34	0.30	3.06	7.05	0.04
16Q	8.44	41.84	27.02	13.24	0.24	2.88	6.29	0.06
24Q	8.01	39.59	31.31	12.05	0.21	2.76	6.00	0.05
				Hours				
1Q	10.70	44.00	20.51	15.05	0.23	1.86	7.19	0.47
4Q	10.15	45.03	23.26	14.48	0.88	3.21	8.72	0.02
8Q	9.14	43.90	27.65	12.97	1.59	2.73	1.88	0.15
16Q	8.01	40.43	34.39	11.21	1.69	2.43	1.69	0.16
24Q	7.53	38.02	37.92	10.88	1.61	2.28	1.61	0.15

Note: 1. η=0.4760, ζ=2.7722

Table 8: Measures of model fit for estimated η and ζ

Fit measure (log value)	η=0.4760, ζ=2.7722
MDD (SWZ)	2632.10
MDD (Mueller)	2625.06
MDD (Bridge)	2624.60

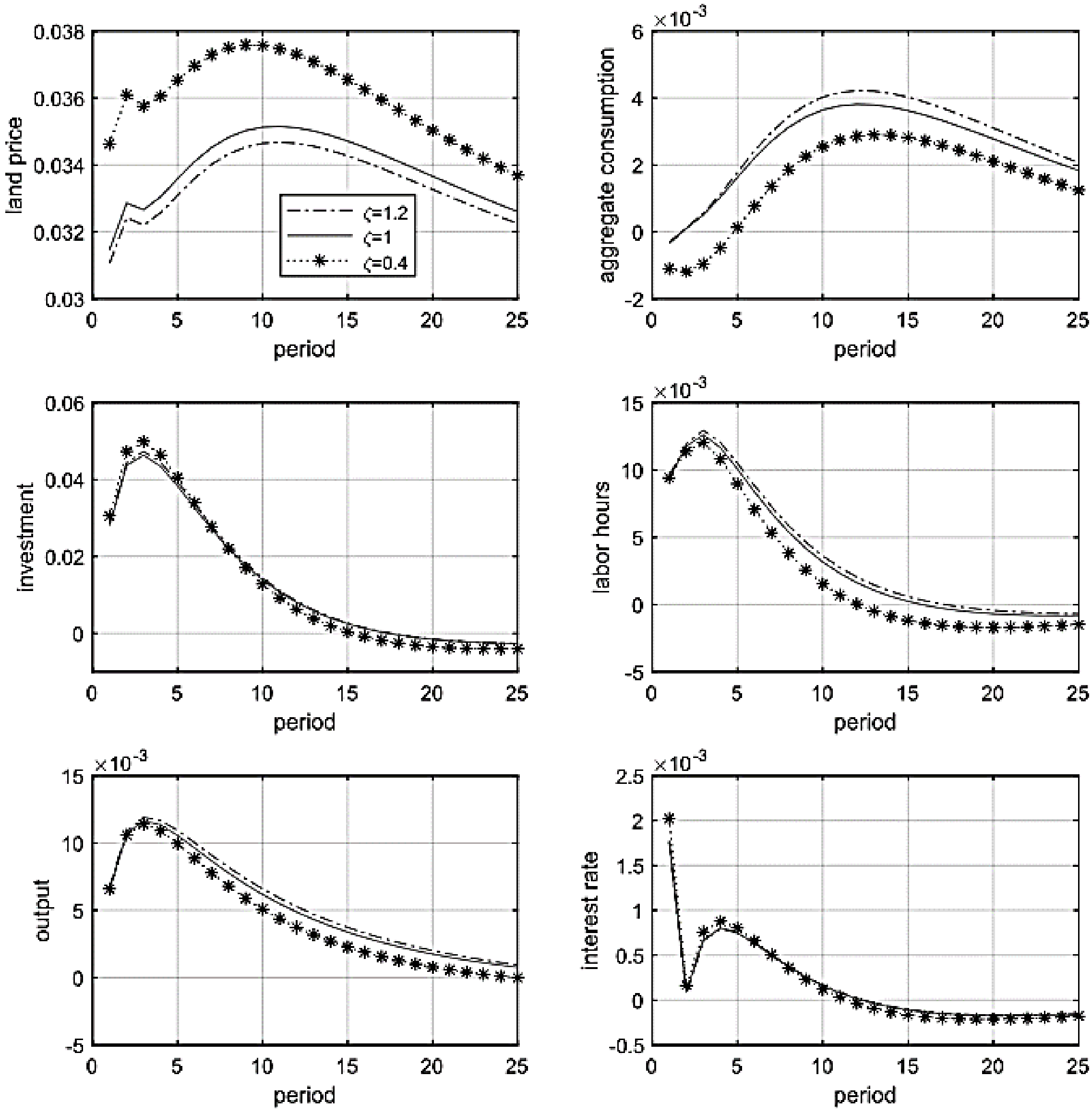


Figure 1. Under baseline η = 1: impulse responses in the cases of ζ = 1.2, 1 and 0.4, respectively, to a positive (one standard deviation) housing demand shock.

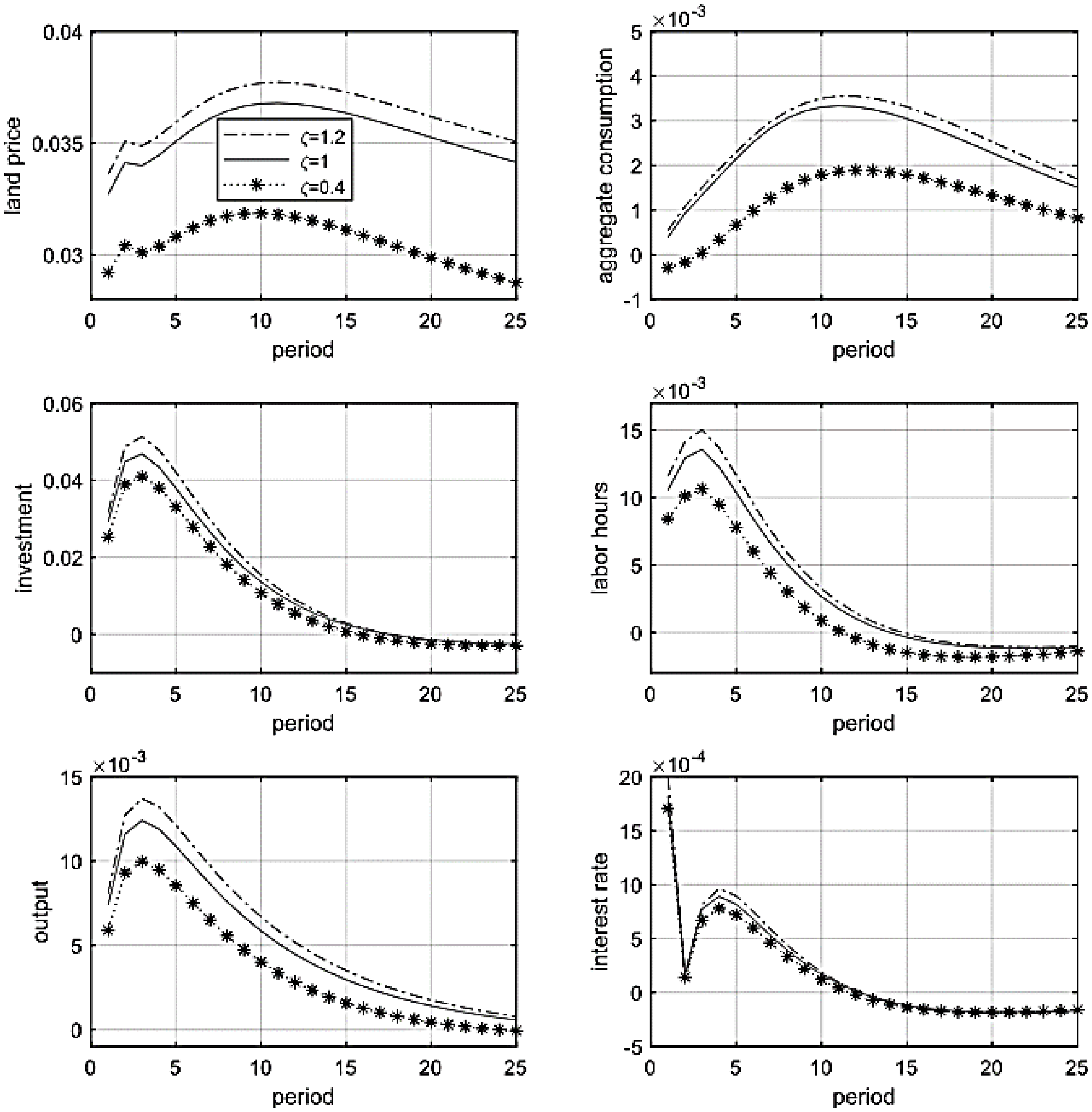


Figure 2. Under baseline η = 0.5, impulse responses in the cases of ζ = 1.2, 1, and 0.4, respectively, to a positive (one standard deviation) housing demand shock.

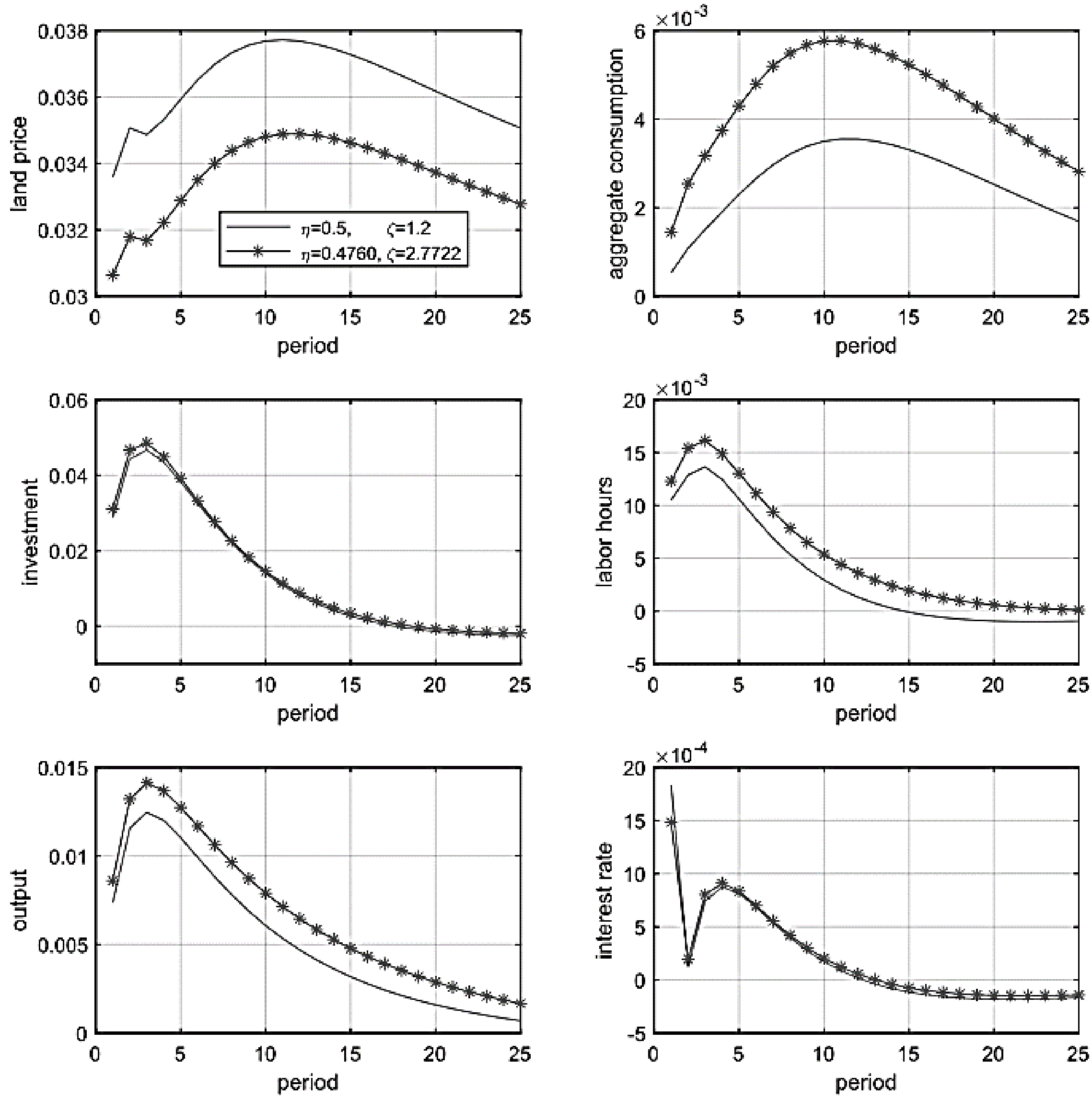
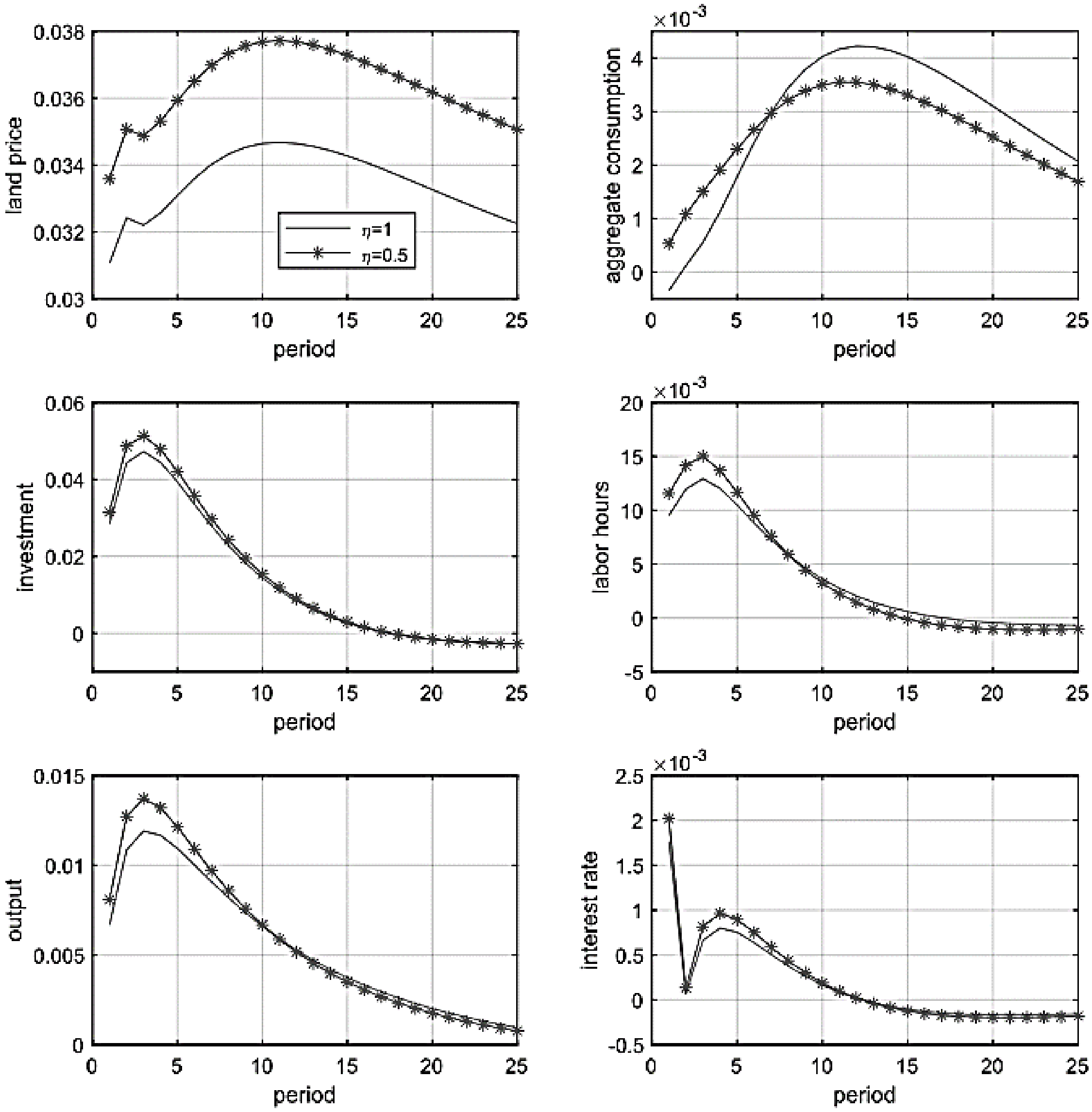
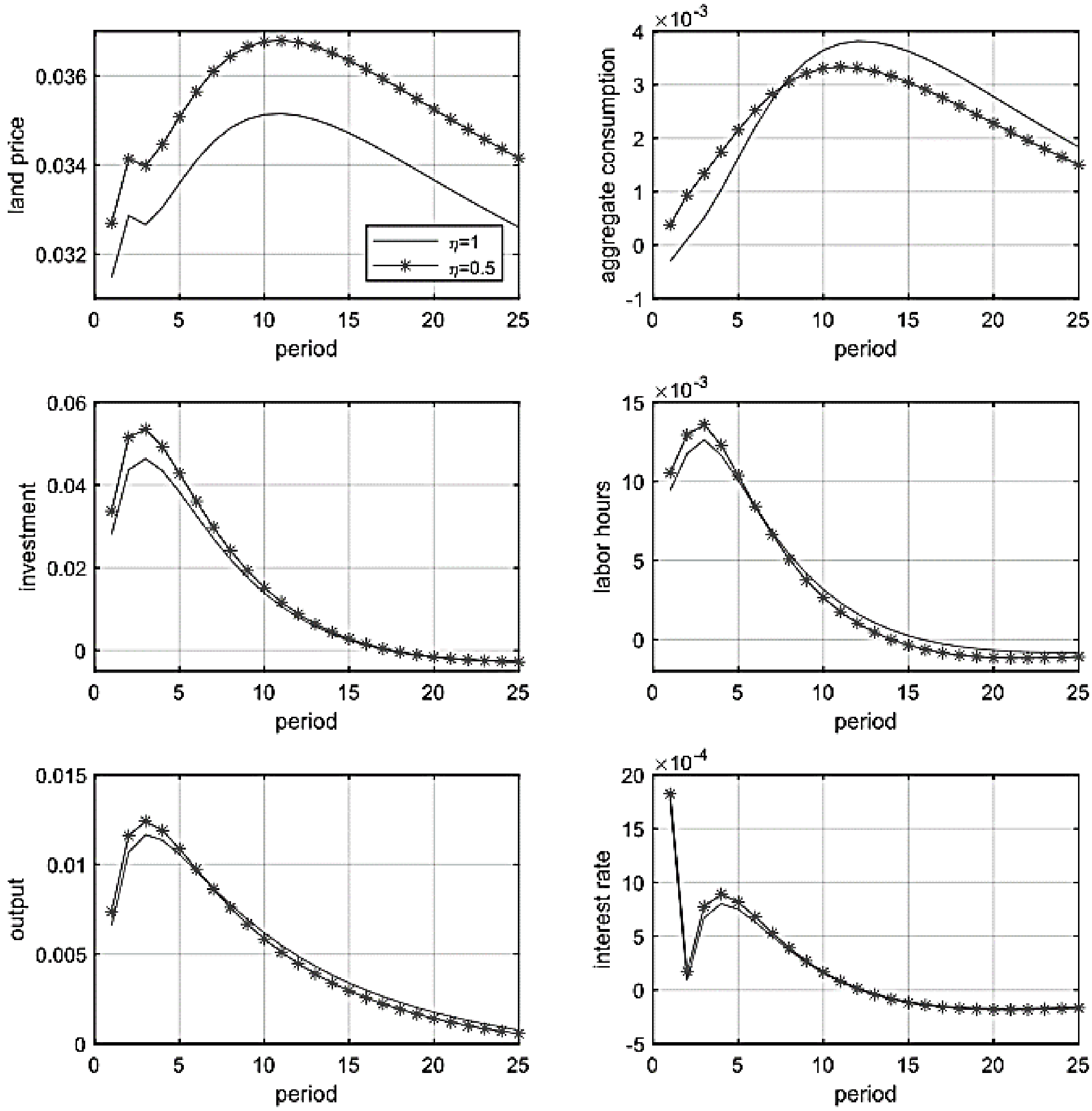


Figure 3. Impulse responses of the model with η=0.4760 and ζ=2.7722 (blue line) and the model with η=05 and ζ=1.2 (red line).

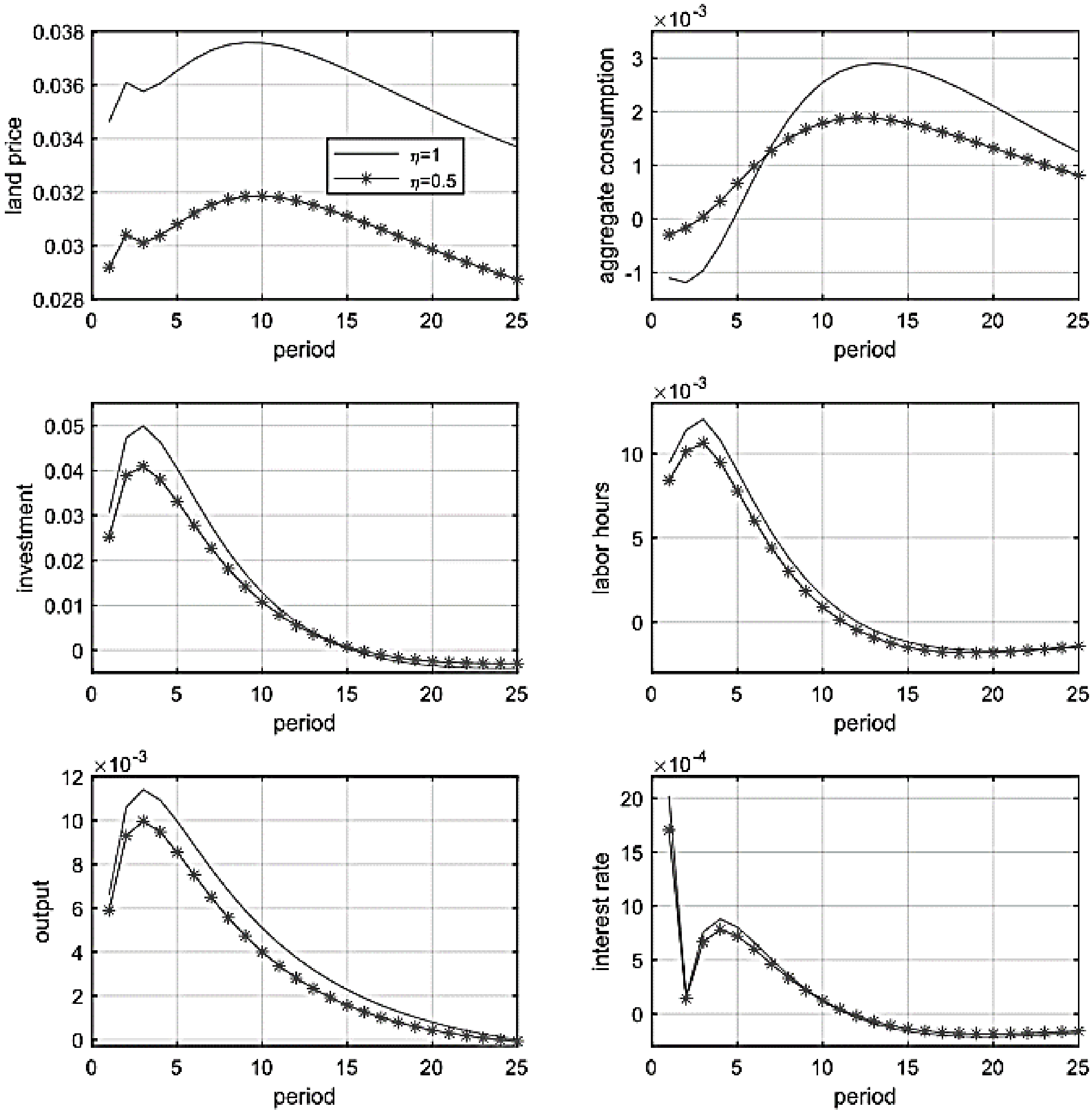
Appendix Figures



Appendix Figure 1. Under ζ = 1.2, impulse responses in the baseline η = 1 (red line) and the baseline η = 0.5 (blue line), respectively, to a positive (one standard deviation) housing demand shock.



Appendix Figure 2. Under $\zeta = 1$, impulse responses in the baseline $\eta = 1$ (red line) and the baseline $\eta = 0.5$ (blue line), respectively, to a positive (one standard deviation) housing demand shock.



Appendix Figure 3. Under $\zeta = 0.4$, impulse responses in the baseline $\eta = 1$ (red line) and the baseline $\eta = 0.5$ (blue line), respectively, to a positive (one standard deviation) housing demand shock.

Mathematical Appendix

Our model generalizes the household utility function in Liu et al. (2013). The household's first order conditions for nondurable consumption and land services and the effects of the elasticity of substitution between consumption and land services are the main differences between our model and Liu et al. (2013). The Appendix is for these two types of differences.

A.1 Derivation of the first order conditions of $C_{h,t}$ and $L_{h,t}$ in the household's problem

$$\max \quad E\sum_{t=0}^{\infty}\beta^t A_t\left[\frac{(u_t)^{\frac{1-1/\eta}{1-1/\zeta}}}{1-1/\eta}-\psi_t N_{h,t}\right],$$

$$\text{s.t. } C_{h,t}+q_{l,t}(L_{h,t}-L_{h,t-1})+\frac{S_t}{R_t}\leq W_t N_{h,t}+S_{t-1},$$

$$\text{where. } u_t\equiv u(C_{h,t},L_{h,t})=(1-\chi)(\tfrac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t})^{1-1/\zeta}+\chi(L_{h,t}^{\varphi_t})^{1-1/\zeta}.$$

Let $\mu_{h,t}$ be the Lagrange multiplier of the budget constraint in period t. First, the first order condition of $C_{h,t}$ is

$$\mu_{h,t}=\tfrac{A_t}{\Gamma_t}u_t^{\frac{1-1/\eta}{1-1/\zeta}-1}(1-\chi)(\tfrac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t})^{\frac{-1}{\zeta}}-\beta E_t\tfrac{A_{t+1}}{\Gamma_{t+1}}\gamma_h u_{t+1}^{\frac{1-1/\eta}{1-1/\zeta}-1}(1-\chi)(\tfrac{C_{h,t+1}-\gamma_h C_{h,t}}{\Gamma_{t+1}})^{\frac{-1}{\zeta}}. \tag{A1}$$

To be consistent with the balanced growth path (BGP), we denote the transformation of a variable consistent with the BGP by the variable with a tilde. Specifically, we denote $\tilde{C}_{h,t}\equiv\frac{C_{h,t}}{\Gamma_t}$, $\tilde{\mu}_{h,t}\equiv\frac{\mu_{h,t}\Gamma_t}{A_t}$, $\tilde{q}_{l,t}\equiv\frac{q_{l,t}}{\Gamma_t}$, where $\Gamma_t\equiv[Z_t Q_t^{(1-\phi)\alpha}]^{1/[1-(1-\phi)\alpha]}$.

Then, condition (A1) can be written in terms of transformed variables as follows.

$$\tilde{\mu}_{h,t}=(1-\chi)\tilde{u}_t^{\frac{\zeta-\eta}{\eta(1-\zeta)}}(\tilde{C}_{h,t}-\tfrac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t-1})^{\frac{-1}{\zeta}}-E_t\beta(1+\lambda_{a,t+1})\tfrac{\gamma_h}{g_{\gamma,t+1}}(1-\chi)\tilde{u}_{t+1}^{\frac{\zeta-\eta}{\eta(1-\zeta)}}(\tilde{C}_{h,t+1}-\tfrac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t})^{\frac{-1}{\zeta}}, \tag{A2}$$

where $\tilde{u}_t=[(1-\chi)(\tilde{C}_{h,t}-\frac{\gamma_h}{g_{\gamma,t}}\tilde{C}_{h,t-1})^{1-\frac{1}{\zeta}}+\chi(L_{h,t}^{\varphi_t})^{1-\frac{1}{\zeta}}]$ and $g_{\gamma,t}\equiv\frac{\Gamma_t}{\Gamma_{t-1}}$.

Let a variable with an upper bar denote the steady state of the variable, and a variable with a hat denote as the variable in a percentage deviation from its steady state. If we take a log-linearization of (A2) around the steady state, we obtain

$$\hat{\mu}_{h,t} = \frac{(1-\chi)\overline{u}^{\frac{1-1/\eta}{1-1/\zeta}-1}(\overline{C}_h - \frac{\gamma_h}{g_\gamma}\overline{C}_h)^{\frac{-1}{\zeta}}}{\overline{\mu}_h}\{[(\tfrac{1-1/\eta}{1-1/\zeta}-1)\hat{u}_t - \tfrac{1}{\zeta}[\tfrac{g_\gamma \hat{C}_{h,t}+\gamma_h(\hat{g}_{\gamma,t}-\hat{C}_{h,t-1})}{g_\gamma-\gamma_h}] - \beta\overline{\lambda}_a \tfrac{\gamma_h}{g_\gamma}\hat{\lambda}_{a,t+1}] \tag{A3}$$

$$+\beta(1+\overline{\lambda}_a)\tfrac{\gamma_h}{g_\gamma}E_t\left(\hat{g}_{\gamma,t+1} - \tfrac{\zeta-\eta}{\eta(1-\zeta)}\hat{u}_{t+1} + \tfrac{1}{\zeta}[\tfrac{g_\gamma \hat{C}_{h,t+1}+\gamma_h(\hat{g}_{\gamma,t+1}-\hat{C}_{h,t})}{g_\gamma-\gamma_h}]\right)\},$$

where

$$\hat{u}_t = \tfrac{1}{\overline{u}}(1-\tfrac{1}{\zeta})\{(1-\chi)\overline{C}_h^{1-1/\zeta}(1-\tfrac{\gamma_h}{g_\gamma})^{-1/\zeta}[\hat{C}_{h,t} + (\gamma_h/g_\gamma)(\hat{g}_{\gamma,t} - \hat{C}_{h,t-1})] + \chi\overline{\varphi}(\overline{L}_h^{\varphi})^{1-1/\zeta}[\hat{L}_{h,t} + (\ln\overline{L}_h)\hat{\varphi}_t]\},$$

$$\hat{\lambda}_{a,t+1} = \tfrac{\lambda_{a,t+1}-\overline{\lambda}_a}{\overline{\lambda}_a},\ \hat{g}_{\gamma,t} = \tfrac{g_{\gamma,t}-\overline{g}_\gamma}{\overline{g}_\gamma},\ \overline{\mu}_h = (1-\chi)[1-\beta(1+\overline{\lambda}_a)\tfrac{\gamma_h}{g_\gamma}]\overline{u}^{\frac{1-1/\eta}{1-1/\zeta}-1}[\overline{C}_h(1-\tfrac{\gamma_h}{g_\gamma})]^{\frac{-1}{\zeta}},\ \text{and}$$

$$\overline{u} = [(1-\chi)(\overline{C}_h - \overline{C}_h\tfrac{\gamma_h}{g_\gamma})^{1-\frac{1}{\zeta}} + \chi(\overline{L}_h^{\varphi})^{1-\frac{1}{\zeta}}].$$

Next, the first order condition of $L_{h,t}$ is

$$\mu_{h,t}q_{l,t} = \beta E_t q_{l,t+1}\mu_{h,t+1} + A_t\frac{\chi\varphi_t L_{h,t}^{\varphi_t(1-1/\zeta)-1}}{[(1-\chi)(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t})^{1-1/\zeta} + \chi(L_{h,t}^{\varphi_t})^{1-1/\zeta}]^{1-\frac{1-1/\eta}{1-1/\zeta}}}. \tag{A4}$$

Condition (A4) is rewritten in terms of transformed BGP variables as follows.

$$\tilde{\mu}_{h,t}\tilde{q}_{l,t} = \beta E_t\left[(1+\lambda_{a,t+1})\tilde{q}_{l,t+1}\tilde{\mu}_{h,t+1}\right] + \chi\varphi_t L_{h,t}^{\varphi_t(1-1/\zeta)-1}\tilde{u}_t^{\frac{1-1/\eta}{1-1/\zeta}-1}. \tag{A5}$$

Taking a log-linearization of (A5) around the steady state give

$$\begin{aligned}\hat{\mu}_{h,t} + \hat{q}_{l,t} = {} & \beta\overline{\lambda}_a E_t\hat{\lambda}_{a,t+1} + \beta(1+\overline{\lambda}_a)E_t(\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}) + \\ & [1-\beta(1+\overline{\lambda}_a)]\{(\tfrac{1-1/\eta}{1-1/\zeta}-1)\hat{u}_t + \hat{\varphi}_t + [\overline{\varphi}(1-1/\zeta)-1]\hat{L}_{h,t} + \overline{\varphi}(1-1/\zeta)(\ln\overline{L}_h)\hat{\varphi}_t\},\end{aligned} \tag{A6}$$

where $\hat{q}_{l,t} = \frac{\tilde{q}_{l,t}-\overline{q}_l}{\overline{q}_l}$ and $\hat{\varphi}_t = \frac{\varphi_t-\overline{\varphi}}{\overline{\varphi}}$.

A.2. Effects of the elasticity of substitution between consumption and land services on land prices

The first order condition of $L_{h,t}$ in (A4) can be rewritten as the following land Euler equation.

$$q_{l,t} = \beta E_t q_{l,t+1}\frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t\Lambda_t(\eta,\zeta)}{\mu_{h,t}}, \tag{A7}$$

where $\Lambda_t(\eta,\zeta) \equiv A_t\chi L_{h,t}^{\varphi_t(1-1/\zeta)-1}[(1-\chi)(\frac{C_{h,t}-\gamma_h C_{h,t-1}}{\Gamma_t})^{1-1/\zeta} + \chi(L_{h,t}^{\varphi_t})^{1-1/\zeta}]^{\frac{1-1/\eta}{1-1/\zeta}-1}$, and the Lagrange multiplier of the budget constraint $\mu_{h,t}$ is in (A1).

The case of Liu et al. (2013) is η=1 and ζ=1, which gives $\Lambda_t(1,1) = \frac{A_t\chi}{L_{h,t}}$, and (A7) reduces to

$$q_{l,t} = \beta E_t q_{l,t+1} \frac{\mu_{h,t+1}}{\mu_{h,t}} + \frac{\varphi_t A_t}{\mu_{h,t}} \frac{\chi}{L_{h,t}}. \tag{A8}$$

So, the difference in terms of the propagation mechanism between our model and the model of Liu et al. (2013) lies in the term $\Lambda_t(\eta, \zeta)$ in (A7). When η=1 but ζ deviates from unity, or when both values of η and ζ deviate from unity, then (A7) is different from (A8). As a result, in response to an increase in the housing demands, the fluctuations in the land price are different, and through the credit constraint, the fluctuations in other macroeconomic variables are different.